Single Sampling Inspection Plans

With Specified Acceptance Probability and Minimum Average Costs.

By

A. Hald.

Best Available Copy

INSTITUTE OF MATHEMATICAL STATISTICS

UNIVERSITY OF COPENHAGEN

December 1964.

Contents.

Prepared with the partial support of the Office of Naval Research (Nonr-N62558-3073). Reproduction in whole or in part is permitted for any purpose of the United States Government.

1. Introduction and summary.

The main purpose of the present paper is to give a tabulation and discussion of properties of a system of single sampling attribute plans obtained by minimizing average costs under the restriction that a point on the OC-curve has been fixed.

Inspection, acceptance, and rejection costs are assumed to be linear in p, the fraction defective, and lot quality is assumed to be distributed according to a double (or as a limiting case a single) binomial distribution with parameters (p_1, p_2, w_1, w_2), $w_1 + w_2 = 1$ and $p_1 < p_2$.

Using average "inspection and sampling costs" as economic unit the standardized average costs become $R = n + (N - n)(\gamma_1 Q(p_1) + \gamma_2 P(p_2))$ where (n, N) denote sample and lot size, respectively, and (γ_1, γ_2) depend on the weights (w_1, w_2) and the decision losses.

Three systems are studied corresponding to different restrictions:

(a) The LTPD system with a fixed consumer's risk, $P(p_2) = 0.10$.

(b) The AQL system with a fixed producer's risk, $Q(p_1) = 0.05$.

(c) The IQL system with $P(p_0) = 1/2$ for $p_1 < p_0 < p_2$.

LTPD and AQL plans for a double binomial prior distribution may be found from the corresponding plans for a single binomial prior distribution by a suitable change of cost parameter.

The solution of the minimization problem and corresponding tables are given for the three systems.

Furthermore the asymptotic properties of the solution are studied.

For the LTPD and AQL systems the main properties of the sampling plans <u>for large N</u> are the following:

(1) Sample size increases linearly with the logarithm of lot size.

(2) The highest allowable fraction defective in the sample converges to the fraction defective with fixed acceptance probability, the difference being of order $1/\sqrt{n}$.

(3) The risk of the producer or the consumer, whichever has not been fixed, tends to zero inversely proportional to lot size.

(4) The minimum costs equal sampling inspection costs plus a constant average decision loss plus a decision loss proportional to $(N - n)$ due to the restriction.

(5) The sampling plans depend only on the product of one cost parameter (being a function of γ_1 and γ_2) and lot size.

For IQL plans both the consumer's and the producer's risk will tend to zero for $N \to \infty$, one of the risks as $O(N^{-1})$ and the other as $O(N^{-1-\delta})$, $\delta \geq 0$. For

$$p_o = \left(\log \frac{q_1}{q_2}\right) \Big/ \left(\log \frac{p_2 q_1}{p_1 q_2}\right)$$

we have $\delta = 0$. The IQL plans are only studied in detail for this value of p_o.

The properties listed under (1),(2), and (5) above are also valid for these IQL plans. Furthermore we have:

(3a) The producer's and the consumer's risks are nearly equal and tend to zero inversely proportional to N.

(4a) The minimum costs equal sampling inspection costs plus a constant average decision loss.

The IQL plans for a double binomial prior distribution may be found with good approximation from the plans for a single binomial prior distribution.

Comparing these plans with the corresponding Bayesian plans the IQL plans have economic efficiency tending to 1 for $N \to \infty$ whereas the efficiency of the LTPD and AQL plans tends to zero.

The restrictions are mainly introduced to obtain protection against deterioration of the prior distribution and because one of the cost components may be (partly) unknown. In such cases it is recommended to use the IQL plans whereas it is not advisable to use the LTPD and AQL plans for large lots. If an upper limit has been specified for the consumer's or the producer's risk one may use the corresponding LTPD or AQL plan for small N and switch over to IQL plans as soon as the condition is satisfied.

The system of sampling plans presented here contains as special cases, viz. for $w_2 = 0$ and $\gamma_1 = 1$, the Dodge-Romig system of LTPD plans, see [2], and the Weibull-Markbäck system of IQL plans, see [13] and [11]. It also contains for $w_2 = 0$ the asymptotic results of a previous paper [7] whereas the tables are different because hypergeometric probabilities were used for the fixed point on the OC-curve in [7] as in the Dodge-Romig tables.

2. The model.

Let N and n denote lot size and sample size and let X and x denote number of defectives in the lot and the sample, respectively. The acceptance number is denoted by c.

Let the costs be

$$nS_1 + xS_2 + (N - n)A_1 + (X - x)A_2 \qquad \text{for } x \leq c$$

and

$$nS_1 + xS_2 + (N - n)R_1 + (X - x)R_2 \qquad \text{for } x > c.$$

The (prior) distribution of X, i.e. the distribution of lot quality, is denoted by $f_N(X)$ and it is assumed that this distribution is a mixed binomial

$$f_N(X) = \int_0^1 \binom{N}{X} p^X q^{N-X} dW(p). \tag{1}$$

In particular $f_N(X)$ may be a <u>double binomial</u>, i.e. a weighted average of two binomials with parameters p_1 and p_2, $p_1 < p_2$, and weights w_1 and w_2, $w_1 + w_2 = 1$. This distribution may also be characterized by saying that p, the process average, has a two-point distribution.

Drawing a sample without replacement from each lot (hypergeometric sampling) and computing the average costs we find

$$K(N,n,c) = \int_0^1 K(N,n,c,p)dW(p) \tag{2}$$

where

$$K(N,n,c,p) = n(S_1 + S_2 p) + (N-n)((A_1 + A_2 p)P(p) + (R_1 + R_2 p)Q(p)), \tag{3}$$

$$P(p) = B(c,n,p) = \sum_{x=0}^{c} \binom{n}{x} p^x q^{n-x}, \tag{4}$$

and $Q(p) = 1 - P(p)$.

For a detailed discussion of this model the reader is referred to Hald [8]. In the following it is assumed that <u>the prior distribution is a double binomial distribution or as a limiting case a single binomial.</u>

To simplify the notation we introduce the three cost functions

$$k_a(p) = A_1 + A_2 p, \qquad k_r(p) = R_1 + R_2 p, \qquad k_s(p) = S_1 + S_2 p, \tag{5}$$

and the averages

$$k_s = w_1 k_s(p_1) + w_2 k_s(p_2) \quad \text{and} \quad k_m = w_1 k_a(p_1) + w_2 k_r(p_2), \tag{6}$$

assuming that $k_s > k_m$. For a double binomial prior distribution k_s represents the average "costs of inspection" per item and k_m represents the average costs per item when all lots from process (component) No. 1 are accepted and all lots from process No. 2 are rejected. As shown in [8] k_m is under certain conditions a useful reference point for average costs per item. Defining the standardized form of (2) as

$$R(N,n,c) = (K(N,n,c) - Nk_m)/(k_s - k_m)$$

it follows that the value of (n,c) minimizing R will also minimize K since k_s and k_m are independent of (n,c). The standardized average costs may be written as

$$R(N,n,c) = n + (N-n)(\gamma_1 Q(p_1) + \gamma_2 P(p_2)) \tag{7}$$

where

$$\gamma_1 = w_1(k_r(p_1) - k_a(p_1))/(k_s - k_m) \text{ and } \gamma_2 = w_2(k_a(p_2) - k_r(p_2))/(k_s - k_m). \qquad (8)$$

The interpretation of (7) is the following: The reduced average "costs of inspection", $k_s - k_m$, have been used as economic unit which means that the total average costs become equal to n, the average costs of inspecting the n sample items, plus the average decision loss per item times the number of items in the remainder of the lot. The term $\gamma_1 Q(p_1)$, say, gives the probability (w_1) of a lot of quality p_1 being submitted (more precisely a lot from a process with process average equal to p_1) times the average probability ($Q(p_1)$) of such a lot being rejected times the corresponding decision loss $((k_r(p_1) - k_a(p_1))/(k_s - k_m))$.

The costs of accepting or rejecting all lots without inspection are $R_a = N\gamma_2$ and $R_r = N\gamma_1$ respectively.

The Bayesian solution of the inspection problem consists in chosing the procedure which leads to the lowest average costs and therefore it requires a comparison of R_a, R_r, and $\min_{(n,c)} R(N,n,c)$. This solution has been discussed and tabulated in [8]. If the Bayesian solution is sampling inspection we shall call the sampling plan minimizing R for the Bayesian (single) sampling plan.

The conditions alluded to above are that $\gamma_1 > 0$ and $\gamma_2 > 0$. For the corresponding Bayesian sampling plan we have $n/N \to 0$, $Q(p_1) \to 0$, and $P(p_2) \to 0$ for $N \to \infty$ which means that $K/N \to k_m$ and $R/N \to 0$ which is one of the reasons for standardizing the average costs in the manner above.

The conditions may also be expressed by means of the economic break-even quality $p_r = (R_1 - A_1)/(A_2 - R_2)$, defined as the root of the equation $k_a(p) = k_r(p)$, since $\gamma_1 > 0$ and $\gamma_2 > 0$ if and only if $p_1 < p_r < p_2$. If $\gamma_1 > 0$ and $\gamma_2 < 0$, say, i.e. $p_r > p_2 > p_1$, the Bayesian solution is acceptance without inspection.

In the present paper we shall consider <u>sampling plans defined by minimizing the average costs under a suitably chosen restriction.</u> The reasons for doing so and the choice of the specific restrictions will be discussed later.

Furthermore we shall also consider cases where <u>either</u> $k_a(p)$ <u>or</u> $k_r(p)$ <u>is identically equal to zero</u> so that γ_2 or γ_1 becomes negative.

One form of restriction is $P(p_o) = 1/2$ for $p_1 < p_o < p_2$. This defines a relation between c and n with the property that $Q(p_1) \to 0$ and $P(p_2) \to 0$ for $n \to \infty$ and consequently $K/N \to k_m$ for $N \to \infty$. Such sampling plans will for the right choice of p_o have similar properties as the Bayesian sampling plans for $\gamma_1 > 0$ and $\gamma_2 > 0$, see section 8.

Another form of the restriction is $P(p_2) = 0.10$, say. This means that $Q(p_1) \to 0$ for $n \to \infty$ and $K/N \to k_m + 0.1\, w_2(k_a(p_2) - k_r(p_2)) = k_m^*$, say. The restriction thus

changes the unavoidable limiting costs from k_m to k_m^* which will therefore be used in standardizing the cost function.

It should be noted, however, that k_m and k_m^* are fundamentally different because k_m depends on the prior distribution and the costs only whereas k_m^* also depends on the restriction which to some extent may be considered arbitrary.

From (7) we find

$$R = n(1-0.1\gamma_2) + (N-n)\gamma_1 Q(p_1) + 0.1\gamma_2 N \tag{9}$$

leading to the (further) standardized costs

$$\frac{R-0.1\gamma_2 N}{1-0.1\gamma_2} = R_o = n + (N-n)\gamma Q(p_1) \tag{10}$$

where $\gamma = \gamma_1/(1-0.1\gamma_2)$. Values of (n,c) minimizing R_o will be the same as those minimizing R.

Similarly we shall use restrictions of the form $Q(p_1) = 0.05$ leading to

$$R_o = n + (N-n)\gamma P(p_2) \tag{11}$$

where $\gamma = \gamma_2/(1-0.05\gamma_1)$.

Restrictions as $P(p_2) = 0.10$ or $Q(p_1) = 0.05$ are of particular interest in cases where $k_a(p)$ or $k_r(p)$, respectively, for some reason has been put equal to zero, i.e. γ_2 or γ_1 becomes negative.

An expression of the type (10) or (11) may, however, be obtained from R by putting $w_2 = 0$ or $w_1 = 0$. It thus follows that <u>a restricted Bayes solution with a two-point prior distribution where the restriction fixes the acceptance probability in one of the two points may be reduced to a restricted Bayes solution with a one-point prior distribution by a suitable change of the cost parameter.</u>

From a mathematical and numerical point of view we may therefore limit ourselves to consider the problem defined by minimizing expressions of the type given by (10) under the restriction stated.

3. Restricted Bayes solutions.

The Bayes procedure has not been widely used in practice for many reasons some of which have been listed below:

(a). It may be difficult to obtain precise information on the prior distributions and the costs.

(b). If the Bayes procedure does not lead to sampling, a running check on the assumptions regarding the prior distribution is lacking.

(c). The mathematical theory behind the Bayes solution is more difficult than for other systems of sampling plans, and adequate tables have been lacking until recently.

With respect to point (b) above it is pointed out that there are <u>two general cases in which the Bayes principle does not lead to a sampling plan</u>, viz. (1) if the prior distribution of p is a one-point distribution or (2) if either $k_a(p) = 0$ or $k_r(p) = 0$.

The first case is important because many investigations have been carried out on the assumption that the quality distribution under "normal conditions" is a binomial distribution. If average quality produced is better than the break-even quality then the cheapest solution will be acceptance without inspection. To obtain a sampling plan minimizing costs under this assumption it is therefore necessary to introduce some sort of restriction.

The second case is important because $k_a(p)$ or $k_r(p)$ are often unknown or may be considered as negligible in the short run when the costs are looked upon from the producer's or the consumer's side exclusively.

One may naturally give up the Bayes solution completely and use the minimax regret solution which depends on the cost parameters only. It seems, however, unreasonable in designing an inspection system for a series of lots from the same source not to use some plausible prior distribution based on existing inspection records and knowledge of normal market quality if only a sampling plan is used in all cases and some insurance has been built into the system against the consequences of a deterioration of the prior distribution and uncertainty in the determination of the cost parameters. This insurance may be formulated in economic terms or in technical terms only and leads to what has been called <u>a restricted Bayes solution</u> since <u>the principle employed is to minimize the average costs under a suitably chosen restriction.</u>

As indicated in section 2 we shall use <u>restrictions which are independent of the weights in the prior distribution and the cost functions</u>. The restrictions considered consist in fixing a point on the OC-curve, i.e. <u>specifying a quality level and a corresponding acceptance probability.</u> Such a restriction defines <u>a relationship between n and c.</u> Restrictions of this kind have first been used by Dodge and Romig [2] in their LTPD system of sampling plans.

The average decision loss depends on expressions of the type $w_2(k_a(p_2)-k_r(p_2))P(p_2)$, say. If we are concerned about the stability of w_2 and the correctness of $k_a(p_2)$ we may get some insurance against consequences of deviations from the values actually used by specifying that $P(p_2)$ shall be small. A detailed discussion of the considerations in connection with fixing a point on the OC-curve will be

given in sections 6-8.

We shall first study LTPD and AQL sampling plans satisfying the restrictions $P(p_2) = 0.10$ and $Q(p_1) = 0.05$, respectively, and thereafter IQL sampling plans satisfying $P(p_o) = 1/2$.

4. The exact solution.

The problem consists in determining (n,c) so that

$$R = n + (N-n)\gamma Q(p_1), \quad \text{(case 1)}, \tag{12}$$

is minimized under the restriction $P(p_2) = P_2$, P_2 being a given number and $p_1 < p_2$, or correspondingly to minimize

$$R = n + (N-n)\gamma P(p_2), \quad \text{(case 2)}, \tag{13}$$

under the restriction $Q(p_1) = Q_1$, $p_1 < p_2$. Since the two problems are of the same mathematical structure we shall discuss only the first one in details.

The problem is similar to Dodge and Romig's problem for the LTPD plans and it will be solved here along similar lines as in Hald [6]. One difference should be noted however, namely that both $Q(p_1)$ and $P(p_2)$ are binomial probabilities, whereas $P(p_2)$ in Dodge and Romig's model is a hypergeometric probability.

To obtain a sampling plan as solution the costs of the plan must be smaller than the costs of complete inspection, i.e. $R < N$, which leads to the condition $Q(p_1) < 1/\gamma$ (case 1) and $P(p_2) < 1/\gamma$ (case 2). It is therefore necessary to assume that $\gamma > 0$ which is also natural from the point of view that γ may be interpreted as the costs per item of rejection or acceptance, respectively, in the case of a one-point prior distribution.

The condition $P(p_2) = B(c,n,p_2) = P_2$ defines a relation between n and c, $n = n_c$ say. Introducing $n = n_c$ in (12) makes R a function of c alone, R(c) say, for any given N. The condition for R(c) to be a local minimum is that

$$\Delta R(c-1) < 0 < \Delta R(c) \tag{14}$$

where $\Delta R(c) = R(c+1) - R(c)$. From (12) we have

$$R(c) = n_c + (N-n_c)\gamma(1-B(c,n_c,p_1))$$

and

$$\begin{aligned}\Delta R(c) &= (1-\gamma)\Delta n_c - N\gamma\Delta B_c + \gamma\Delta(n_c B_c) \\ &= (1-\gamma+\gamma B_c)\Delta n_c - \gamma(N-n_{c+1})\Delta B_c \end{aligned} \tag{15}$$

where $B_c = B(c,n_c,p_1)$.

Introducing the auxiliary function

$$N_c = \frac{(1-\gamma)\,\Delta n_c + \gamma\,\Delta (n_c B_c)}{\gamma\,\Delta B_c} = n_{c+1} + \left(\frac{1}{\gamma} - (1-B_c)\right)\frac{\Delta n_c}{\Delta B_c}\ , \tag{16}$$

substituting (15) into (14), and "solving" for N lead to the fundamental inequality

$$N_{c-1} < N < N_c \tag{17}$$

together with $\Delta B_{c-1} > 0$ and $\Delta B_c > 0$ as the conditions for (n_c, c) to be the optimum plan for lot size N.

In case 2 the corresponding result is

$$N_c = n_{c+1} + \left(\frac{1}{\gamma} - B_c\right)\frac{\Delta n_c}{\Delta(1-B_c)} \tag{18}$$

together with $\Delta(1-B_{c-1}) > 0$ and $\Delta(1-B_c) > 0$ where $B_c = B(c,n_c,p_2)$ and $B(c,n_c,p_1) = 1-Q_1$.

It has only been proved that (17) is the condition for R(c) to be a local minimum. A similar analysis may, however, be carried out by means of the difference operator $\Delta_i R(c) = R(c+i)-R(c)$. The condition for R(c) to be an absolute minimum is that $\Delta_i R(c) > 0$ for $i = 1,2,\ldots,n-c$, and $\Delta_i R(c-i) < 0$ for $i = 1,2,\ldots,c$. It is easily seen that sufficient conditions for these inequalities to be fulfilled are that R(c) be a local minimum, i.e. (17) is fulfilled, and furthermore that N_c be a non-decreasing function of c, since the inequalities $N < N_c \le N_{c+1} \le \ldots \le N_{c+i-1}$ by addition of all the numerators and denominators lead to

$$N < \frac{(1-\gamma)\,\Delta_i n_c + \gamma\,\Delta_i (n_c B_c)}{\gamma\,\Delta_i B_c} \tag{19}$$

i.e. $\Delta_i R(c) > 0$ for $i > 0$. It is conjectured that N_c is a non-decreasing function of c if n_c is considered as a continuous variable. However, in tabulating the solution only integer values of n_c has been used which means that the condition $P(p_2) = P_2$(and the other similar conditions) will in most cases not be exactly fulfilled. For the three cases tabulated the cumulative binomial has been computed to six decimal places and $n = n_c$ has been determined as

(1) the smallest integer n satisfying $B(c,n,p_2) \le 0.10$,

(2) the integer n for which $B(c,n,p_0)$ is nearest to 0.50,

(3) the largest integer n satisfying $B(c,n,p_1) \ge 0.95$.

If N_c is an increasing function of c it follows from (17) that for each (c,n_c) there exists an "optimum interval "(N_{c-1},N_c) so that for all N within that interval the optimum plan is (c,n_c). In case $N_c < N_{c-1}$ the plan (c,n_c) is not optimum for any N and has to be excluded. The costs R(c-1) and R(c+1) have then to be compared.

Using $R(c+1) - R(c-1) = \Delta R(c) + \Delta R(c-1)$ it follows that $R(c-1) \lesseqgtr R(c+1)$ for $N \lesseqgtr N^*_{c-1}$ where

$$N^*_{c-1} = (N_{c-1}\Delta B_{c-1} + N_c \Delta B_c)/(\Delta B_{c-1} + \Delta B_c).$$

In that manner the optimum plans and the corresponding N-intervals may successively be determined starting from c = 0. The procedure is well suited for an electronic computer. The tables will be discussed in the following sections.

For large N the Poisson distribution may be used as an approximation to the binomial. The original problem may also be such that the Poisson distribution is the appropriate one to use, viz. if quality is measured in number of defects per unit instead of in fraction defective. For these reasons the Poisson solution has also been tabulated. First $m = m_c$ has been determined from the relation

$$B(c,m) = \sum_{x=0}^{c} e^{-m} \frac{m^x}{x!} = P_2, \quad m = np_2 \quad . \tag{20}$$

The inequality corresponding to (17) becomes

$$M_{c-1} < M < M_c, \quad M = Np_2, \tag{21}$$

where

$$M_c = \frac{(1-\gamma) \Delta m_c + \gamma \Delta (m_c B_c)}{\gamma \Delta B_c} = m_{c+1} + (\frac{1}{\gamma} - (1-B_c))\frac{\Delta m_c}{\Delta B_c} \tag{22}$$

and

$$B_c = \sum_{x=0}^{c} e^{-rm_c} \frac{(rm_c)^x}{x!}, \quad r = \frac{p_1}{p_2} \quad . \tag{23}$$

Since $m = np_2$ is a function of c only whereas in the binomial case m is a function of both p_2 and c, it is possible to give a much more compact tabulation of the Poisson solution than of the binomial.

For small values of N the solution given above need to be modified in certain cases.

For $N \leq n_0$ no sampling plan exists satisfying the restriction required. In such cases the solution has been given as "all" in the tables to indicate that inspection of the whole lot is necessary to obtain a protection as least as good as the one required.

In case 1 for $\gamma \geq 1$ the alternative to sampling inspection is total inspection which costs N. To obtain R < N it is necessary that $Q(p_1) < 1/\gamma$, i.e. $B_c > 1-1/\gamma$, which may not be fulfilled for small c and corresponding values of $N \in (N_{c-1}, N_c)$. In such cases the cheapest sampling plan available has nevertheless been given in the table but a "*" has been added to indicate that sampling is more costly than total inspection.

In case 1 for $\gamma < 1$ the alternative to sampling inspection is rejection at a cost of $N\gamma$. To obtain $R < N\gamma$ it is necessary that

$$N > n_c(1 + \frac{1-\gamma}{\gamma B_c}), \quad B_c = 1-Q(p_1). \tag{24}$$

The corresponding result in case 2 for $\gamma \geq 1$ is $P(p_2) < 1/\gamma$, i.e. $B_c < 1/\gamma$, and for $\gamma < 1$ with acceptance as alternative $R < N\gamma$ which leads to

$$N > n_c \ (1 + \frac{1-\gamma}{\gamma(1-B_c)}) \ , \ B_c = P(p_2). \tag{25}$$

In such cases "a" has been added after the sample size to indicate that acceptance without inspection is cheaper than sampling.

It has furthermore to be taken into account that (c, n_c) may be used as optimum plan for N only if $N_{c-1} < N \leq N_c$ and $N > n_c$. If $N_c < N \leq n_{c+1}$ no optimum plan exists because (c, n_c) is not optimum for $N > N_c$ and $(c+1, n_{c+1})$ cannot be used because $N \leq n_{c+1}$. It is therefore a condition for the existence of optimum plans that $N_c > n_{c+1}$. From (16) and (18) follows, however, that this condition may be reduced to the one following from $R < N$.

5. The asymptotic solution.

The procedure in arriving to an asymptotic solution giving c and n as explicit functions of N will be first to get an asymptotic expansion of c in terms of n as an expression for the condition imposed and then to eliminate c from R and solve the equation $dR/dn = 0$ after having replaced the binomial probability in R by an asymptotic expansion in terms of n. A similar method has been used in [6].

A rather accurate solution of the equation $B(c,n,p) = P$ may be obtained by using the expansion of Fisher and Cornish [5] which leads to

$$c = np + u_P \sqrt{npq} + \frac{1}{6}(q-p)(u_P^2-1) - \frac{1}{2} + O(n^{-\frac{1}{2}}) \tag{26}$$

where u_P denotes the P-fractile of the standardized normal distribution.

Writing $h = c/n$ the condition $P(p_2) = P_2$ may therefore be expressed as

$$h = p_2 + a\sqrt{p_2 q_2 / n} + b/n + O(n^{-\frac{3}{2}}) \tag{27}$$

where

$$a = u_{P_2} \text{ and } b = \frac{1}{6} (q_2 - p_2)(a^2-1) - \frac{1}{2} \ . \tag{28}$$

Since $h = p_2 + O(n^{-\frac{1}{2}})$ we may use the following lemma which is a special case of a theorem proved by Blackwell and Hodges [1]:

For $p_1 < p_2$ and $n \to \infty$ we have

$$1 - B(c,n,p_1) = \frac{1}{\sqrt{2\pi m p_2 q_2}} \; \frac{q_2 p_1}{p_2 - p_1} \; e^{-n\varphi(h,p_1)} (1 + O(n^{-\frac{1}{2}})) \tag{29}$$

where

$$\varphi(h,p) = h \ln \frac{h}{p} + (1-h) \ln \frac{1-h}{1-p} . \tag{30}$$

(For $h = p_1 + O(n^{-\frac{1}{2}})$ a similar expression is valid for $B(c,n,p_2)$).

Setting

$$f(n) = \frac{\lambda}{\sqrt{n}} e^{-n\varphi(h,p_1)} \tag{31}$$

and

$$\lambda = \frac{1}{\sqrt{2\pi p_2 q_2}} \; \frac{\gamma q_2 p_1}{p_2 - p_1} \tag{32}$$

we find from (12) and (29)

$$R = n + (N-n) f(n) (1 + O(n^{-\frac{1}{2}})) . \tag{33}$$

Expanding $\varphi(h,p_1)$ in a Taylor series around p_2 and inserting $h-p_2$ from (27) we get

$$\begin{aligned} \varphi(h,p_1) &= \varphi(p_2,p_1) + (h-p_2) \ln \frac{p_2 q_1}{p_1 q_2} + \frac{1}{2 p_2 q_2} (h-p_2)^2 + O(n^{-\frac{3}{2}}) \\ &= \varphi(p_2,p_1) + a\sqrt{\frac{p_2 q_2}{n}} \ln \frac{p_2 q_1}{p_1 q_2} + \frac{1}{n} \left(\frac{a^2}{2} + b \ln \frac{p_2 q_1}{p_1 q_2}\right) + O(n^{-\frac{3}{2}}) . \end{aligned} \tag{34}$$

It follows that $\sqrt{n} f(n)$ tends exponentially to zero for $n \to \infty$ since $\varphi(p_2,p_1) > 0$.

From (33) we find

$$\frac{dR}{dn} = 1 + (N-n) f'(n) - f(n) .$$

Solving the equation $dR/dn = 0$ for $N-n$ we get

$$N-n = -\frac{1}{f'(n)} (1 + O(n^{-\frac{1}{2}}))$$

since $f'/f \to -\varphi(p_2,p_1)$ and $f \to 0$.

Writing

$$\ln(N-n) = -\ln f(n) - \ln(-f'(n)/f(n)) + O(n^{-\frac{1}{2}}) \tag{35}$$

we finally have

$$\ln(N-n) = \alpha_1 n + \alpha_2 \sqrt{n} + \frac{1}{2} \ln n + \alpha_3 + O(n^{-\frac{1}{2}}) \tag{36}$$

where $\alpha_1 = \varphi(p_2,p_1)$, $\alpha_2 = a\sqrt{p_2 q_2} \ln (p_2 q_1 / p_1 q_2)$, and

$$\alpha_3 = a^2/2 + b \ln (p_2 q_1 / p_1 q_2) - \ln \lambda - \ln \varphi(p_2,p_1) .$$

The same formula applies to case 2 if only p_1 and p_2 are interchanged, (p_2-p_1 in (32) should be read as $|p_2-p_1|$), and P_2 in (28) is replaced by $P_1 = 1-Q_1$.
This result is a generalization of the one obtained in [6] partly because it is based on the binomial instead of the Poisson distribution and partly because the model here contains a cost parameter γ which is equal to 1 in the case previously considered.

Solving (26) with respect to np gives

$$np = c + 1 - u_p\sqrt{(c+1)q} + (u_p^2 - 1)(1+p)/3 - u_p^2 p/2 + O(c^{-\frac{1}{2}}). \tag{37}$$

Formulas (36) and (37) give good approximations to the exact solution for $Np_2 > 15$, $p_1/p_2 \leq 0.5$, and $P_2 = 0.10$ or 0.50, and in case 2 for $Np_1 > 15$, $p_2/p_1 \geq 1.5$, and $Q_1 = 0.05$ or 0.50.

The formulas should be used as follows: For c = 0.5, 1.5, 2.5,... n is computed from (37) and N from (36) to obtain <u>intervals</u> for N corresponding to every integer value of c, cf. (17). For each integer value of c the appropriate sample size is determined from (37).

A formula giving the sample size directly as function of lot size may be obtained by inversion of (36) which according to the result given in [6] leads to

$$n = \beta_1 x + \beta_2\sqrt{x} + \beta_3 \ln x + \beta_4 + \beta_5(\ln x)/\sqrt{x} + \beta_6/\sqrt{x} \tag{38}$$

where $x = \ln N$, $\beta_1 = 1/\alpha_1$, $\beta_2 = -\alpha_2/\alpha_1^{3/2}$, $\beta_3 = -\beta_1/2$, $\beta_4 = (\ln \alpha_1 + \alpha_2^2/\alpha_1 - 2\alpha_3)/2\alpha_1$, $\beta_5 = -\beta_2/4$, and $\beta_6 = \beta_5(2 - 2\alpha_1\beta_4 + \alpha_2^2/2\alpha_1)$.

For a given N we may compute n by (38) and the corresponding c by (26), round c to the nearest integer and find n from (37).

Numerical investigations have shown that (38) leads to rather accurate results for $P_2 = 0.10$, $p_2 \leq 0.10$, $p_1/p_2 \leq 0.5$, and $Np_2 > 15$, whereas it should not be used for $P_2 = 0.50$ or in case 2 for $Q_1 = 0.05$.

From (35) it follows that

$$\ln(N-n)f(n) = -\ln \varphi(p_2,p_1) + O(n^{-\frac{1}{2}})$$

or

$$(N-n)\gamma\, Q(p_1) = \frac{1}{\varphi(p_2,p_1)}(1 + O(n^{-\frac{1}{2}})) \tag{39}$$

and consequently

$$\min_{(n,c)} R = n + \frac{1}{\varphi(p_2,p_1)} + O(n^{-\frac{1}{2}}) \tag{40}$$

where n is given by (38).

We have thus found the following <u>asymptotic properties of the solution:</u>

(1) Sample size increases linearly with the logarithm of lot size, see (38).

(2) The highest allowable fraction defective in the sample converges to the fraction defective with fixed acceptance probability, the difference being of order $1/\sqrt{n}$, see (26).

(3) The risk of the producer or the consumer, whichever has not been fixed, tends to zero inversely proportional to lot size, see (39).

(4) The minimum (standardized) costs equal sampling inspection costs plus a constant depending on (p_1, p_2) only, see (40).

Analogous results have previously been given by Hald [6] for the case with $\gamma = 1$ and Poisson probabilities.

The last mentioned property means that asymptotically decision losses will be negligible as compared to sampling inspection costs.

This is true, however, only for cost functions of form $R_o = n + (N-n)\gamma Q(p_1)$. If we have a cost function as (7) then $R = (1-0.1\gamma_2)R_o + 0.1\gamma_2 N$ which asymptotically equals

$$\underset{(n,c)}{\text{Min}}\, R = (1 - 0.1\gamma_2)(n + \frac{1}{\varphi(p_2, p_1)}) + 0.1\gamma_2 N$$

where the first term is $O(\ln N)$. For large N the term $0.1\gamma_2 N$ resulting from the restriction $P(p_2) = 0.1$ becomes dominating in contrast to the result for the (unrestricted) Bayesian sampling plan where min $R = O(\ln N)$, see [8]. For $\gamma_2 > 0$ the economic efficiency of a restricted Bayesian sampling plan of the type above as compared to a Bayesian plan will thus tend to zero for $N \to \infty$. For $\gamma_2 < 0$ the Bayesian solution is acceptance without inspection at a cost of $R_a = N\gamma_2$.

The asymptotic formulas also reveal that the cost parameter γ influences the solution in an extremely simple manner. From (36) it will be seen that γ only enters through a_3 so that $\ln(N\gamma) = F(n)$ where $F(n)$ is independent of γ. It follows that <u>asymptotically the sampling plan only depends on the product of lot size and cost constant</u> so that for example the plan for lot size N and cost constant γ equals the plan for lot size $N\gamma$ and cost constant 1.

Since this property holds for large γ-intervals also for small values of N it is only required to tabulate sampling plans for rather few values of γ.

Consequently it should be noted that the Dodge-Romig LTPD tables may be used to find sampling plans by entering the tables with $N^* = N\gamma$ for $\gamma < 3$ say.

Another way of expressing the dependence of γ is given by

$$n(N,\gamma) \sim n(N,1) + \frac{\ln \gamma}{\varphi(p_2,p_1)} \tag{41}$$

which follows from (38).

Formula (39) shows another interesting result, viz. that the asymptotic value of $Q(p_1)$ is inversely proportional to γ, which is the reason that min R only depends on γ through n, see (40).

6. LTPD sampling inspection plans with minimum costs.

LTPD plans are here defined as sampling plans with a given Lot Tolerance Per Cent Defective, $100p_2$, and a corresponding probability of acceptance, the consumer's risk $P(p_2)$, which traditionally is chosen as 10 per cent.

In the discussion of sampling plans it has been found convenient for obvious terminological and pedagogical reasons to introduce a fictitious consumer and producer and concentrate attention on the corresponding two points on the OC curve, $P(p_1)$ and $P(p_2)$, $p_1 < p_2$, defining the producer's risk as $Q(p_1)$ and the consumer's risk as $P(p_2)$. It is useful to extend these notions also to the cost functions.

Consider a producer inspecting his own product before delivery and suppose that he has essentially two goals: (1) To make reasonably sure that lots of bad quality are not marketed. (2) To keep his inspection costs and decision losses down.

We shall in turn discuss these aspects of the problem under two different assumptions regarding the prior distribution, viz. for a one-point and a two-point distribution of p.

Suppose that the producer knows his process average p_1 for "normal production" and that he occasionally produces lots of bad quality. The quality level for bad lots may be fluctuating rather much so that the producer is not willing neither to state an average quality level for these lots nor the frequency with which such lots will occur. However, the producer may be willing to select a tolerance value of the fraction defective, p_2 say, and a risk, $P(p_2)$, of accepting lots of this quality. The choice of p_2 is difficult and rather subjective. It is based on considerations of customary market quality, the producer's own quality performance, his prestige, consequences of loss of good-will, consequences for the consumer of getting bad quality, the use of the product, etc. The consumer's risk, $P(p_2)$, is customarily chosen as 0.10. This is perfectly arbitrary and the value of $P(p_2)$ has therefore to be kept in mind in choosing p_2 even if ideally p_2 should be determined exclusively from technical and economical considerations.

Turning now to the costs the first question to be answered is the following: What are the producer's average costs for lots of normal quality ? The answer is given by the value of the cost function $K(N,n,c,p_1)$, see (3). In many cases, however, it seems reasonable to disregard the term $(A_1+A_2p_1)P(p_1)$ from the producer's point of view because lots of quality p_1 are supposed to be satisfactory as general market quality or by mutual (tacit) agreement between the parties. Delivery of lots of quality p_1 will therefore not lead to (essential) complaints from the consumer, i.e. the consumer has to bear the costs due to accepted defective items. If this is so one may merely put $A_1 = A_2 = 0$ in the following formulas.

Since the producer cannot specify the quality level and the frequency of bad lots it is impossible to include the corresponding costs in the discussion. A low frequency of bad lots and the restriction $P(p_2) = 0.10$ should, however, if p_2 has been chosen sufficiently small, make sure that very few bad lots will be accepted so that no serious economic damage will result.

Under these circumstances it seems therefore reasonable to determine the sampling plan by minimizing the producer's costs for lots of normal quality, $K(N,n,c,p_1)$, under the restriction of a fixed consumer's risk, $P(p_2) = 0.10$. From the point of view of statistical theory this is a restricted Bayes solution with a one-point prior distribution of p.

Introducing the standardized costs

$$R = (K(N,n,c,p_1) - Nk_a(p_1))/(k_s(p_1) - k_a(p_1))$$

we find

$$R = n + (N-n)\gamma_1 Q(p_1)$$

with

$$\gamma_1 = \frac{k_r(p_1) - k_a(p_1)}{k_s(p_1) - k_a(p_1)} = \frac{R_1 - A_1 - (A_2 - R_2)p_1}{S_1 - A_1 - (A_2 - R_2)p_1}, \tag{42}$$

see (7) for $w_2 = 0$. This shows that the solution is the one discussed in sections 4 and 5 with cost parameter equal to γ_1 (for $w_1 = 1$).

The solution only requires knowledge of the two quality levels and the cost constants. It rests on the assumption that the quality distribution of the larger part of the lots is a binomial distribution. A weakness is the uncertainty in the determination of p_2 and $P(p_2)$. For practical reasons it is customary to use $P(p_2) = 0.10$ in constructing tables of the solution. The parameter left free in practice is therefore p_2 only (p_1 is assumed to be rather accurately known by the producer) and since sample size is a decreasing function of p_2 for given p_1, the producer may in case of doubt choose a small value of p_2 which will lead to a sharper discrimination between good and bad lots.

This system of sampling plans is a generalization of the Dodge-Romig LTPD system which may be obtained for $\gamma_1 = 1$. Dodge and Romig assume that rejection means complete inspection of the remainders of rejected lots and furthermore that the costs of complete inspection per item are the same as the costs of sampling inspection, i.e. $k_r(p) = k_s(p)$. The cost parameter γ_1 allows us to interprete "rejection" in a much wider sense than Dodge and Romig and also to take costs of acceptance into account if necessary, see the definition of γ_1 in (42). It should be noted that the consumer's risk in the tables given here has been computed as a binomial probability whereas in the previous paper [7] the hypergeometric distribution has been used as in Dodge and Romig's tables.

Suppose now that submitted lots are distributed according to a double binomial distribution with parameters (p_1, p_2, w_2). If the parameters are known and the distribution is stable and if $p_1 < p_r < p_2$ the Bayes solution may be determined as in [8]. If, however, the stability of the prior distribution is doubtful and/or information on costs is incomplete the producer may prefer a restricted Bayes solution.

Firstly the producer may find it necessary to protect himself against some of the consequences of undesirable (and unknown) changes of the prior distribution and for that reason he may impose the condition $P(p_2) = 0.10$ on the plans.

Secondly the costs of acceptance may be partly unknown, e.g. because loss of goodwill is involved. In the short run the producer may regard costs of acceptance as practically negligible, i.e. $k_a(p) = 0$, if the consumer does not return an occasional bad lot but (possibly) only bad items found. If bad lots are returned by the consumer we have $k_a(p) = k_r(p)$ plus costs of delivering and returning the lots. In the long run, however, bad lots delivered will result in loss of goodwill which may be difficult to evaluate and include explicitly into the cost function.

The producer may therefore be forced to modify the model. Instead of minimizing the complete cost function without any restrictions he may choose to minimize the incomplete cost function obtained by putting $k_a(p) = 0$ under the restriction $P(p_2) = 0.10$ hoping that the resulting small frequency of bad lots accepted will reduce his (unknown) costs of acceptance sufficiently. As indicated above there may be cases where it is more reasonable to put $k_a(p_1) = 0$ and $k_a(p_2) = k_r(p_2)$.

One of the effects of fixing $P(p_2)$ may be judged by noting that on the average the ratio of number of bad lots accepted to total number of lots accepted will be $w_2P(p_2)/(w_1P(p_1) + w_2P(p_2))$. For $P(p_2) = 0.10$ and $P(p_1) \approx 1$ this ratio will be 1/191 for $w_2 = 0.05$ and 1/91 for $w_2 = 0.10$.

The sampling plan is determined by minimizing the average costs, $K(N,n,c)$, under

the restriction of a fixed consumer's risk, $P(p_2) = 0.10$. It follows from (10) that the solution has been given in sections 4 and 5 for the following value of the cost parameter

$$\gamma = \frac{w_1(k_r(p_1) - k_a(p_1))}{w_1(k_s(p_1) - k_a(p_1)) + w_2(k_s(p_2) - 0.1k_a(p_2) - 0.9k_r(p_2))} = \frac{\gamma_1}{1 - 0.1\gamma_2} \tag{43}$$

where γ_1 and γ_2 are defined by (8).

For $w_2 = 0$ we have $\gamma = \gamma_1$ which means that from a mathematical point of view the approach leading to (42) may be regarded as a limiting case of the one above. As will be explained later in this section the same tables may therefore be used to obtain the optimum sampling plan in both cases.

It should be noted, however, that the interpretation of p_2 is different. In the first case p_2 is a tolerance fraction defective determined from technical and economical considerations whereas in the second case p_2 is a parameter in the prior distribution, viz. the average fraction defective for lots of unsatisfactory quality.

For $w_2 > 0$ the sign of γ_2 determines whether γ is smaller or greater than γ_1. If $k_a(p) = 0$ then $\gamma_2 < 0$ and $\gamma < \gamma_1$.

In case w_2 is known only approximately but limits for w_2 may be guessed at then max γ can be found and used to get an upper limit for the appropriate sample size. Similarly, if p_2 is known only approximately max γ can be found by chosing p_2 as small as reasonable.

It is important to notice that normally the second term of the denominator of (43) is negligible as compared to the first so that γ_1 may be used as a good approximation to the cost parameter which again means that in important practical cases ($k_a(p) = 0$, $k_r(p) = R_1$, and $k_s(p) = S_1$) γ will approximately be equal to the ratio between the costs of rejection per item and the costs of sampling inspection per item. This may be seen in the following way. For $k_a(p) = 0$ we find $\gamma = w_1k_r(p_1)/(k_s - 0.9w_2k_r(p_2))$. If furthermore $k_r(p) = R_1$ and $k_s(p) = S_1$ we have

$$\gamma = \frac{R_1}{S_1} \Bigg/ \left(1 + \frac{w_2}{w_1}\left(1 - 0.9\frac{R_1}{S_1}\right)\right). \tag{44}$$

For $R_1 = S_1$ we find $\gamma = w_1/(w_1 + 0.1w_2)$. This shows that the sampling plan is rather insensitive to changes of w_2 unless R_1/S_1 is large.

The above discussion of the LTPD system has been carried out from a producer's point of view. For positive values of (γ_1, γ_2) similar considerations may be made by a consumer.

As mentioned in section 4 tables may be constructed with c as argument and (n,N) as functions of c. Such tables have, however, only been given for the solution based on Poisson probabilities because in that case it suffices to tabulate $m = np_2$ and $M = Np_2$ as functions of c for a given value of $r = p_1/p_2$, see (20)-(23), which makes it possible to set up a compact and rather complete table. The two functions have been tabulated for 14 values of r (0.05, 0.10,...,0.70) and for $c \leq 99$ with the modification that tabulation has been stopped when M exceeds 50,000. Because only an abridged version is published the last figure for M given in a column may be less than 50,000 even if $c < 99$ which means that M exceeds 50,000 for the next entry. M has been determined to three significant figures. The optimum plan is (c,m) for $M_{c-1} < M < M_c$. For $\gamma = 5$ some of the smaller values of M have been underlined to indicate that total inspection for these values of M is cheaper than sampling inspection, and that the plan tabulated is the cheapest sampling plan available, i.e. $M_c = m_{c+1}$.

For practical reasons, i.e. to save space and to make the tables easier to use in practice, the tables based on binomial probabilities give (n,c) as functions of N. The exact solution, derived as described in section 4, has been given for $100p_2 = 0.5,1,2,3,4,5,7,10,15,20$, for five values of $r = p_1/p_2$ chosen among the values r = 0.1,0.2,...,0.7, and for $\gamma = 1$ and 5, giving a total of 10 x 5 x 2 = 100 tables. The same 20 values of N between 30 and 200,000 have been used in all the tables. Plans have been computed only for $c \leq 99$. The tables also contain $P(p_1)$ which makes it easy to compute $R_o = n + (N-n)\gamma Q(p_1)$, $R = (1-0.1\gamma_2)R_o + 0.1\gamma_2 N$, and the average costs $K = (k_s - k_m)R + Nk_m$.

For $\gamma > 1$ it may happen that total inspection is cheaper than sampling inspection for small lots. The cheapest sampling plan available (c as large as possible) has nevertheless been tabulated, and the letter <u>t</u> (for total inspection) has been added after the sample size. Such samples will be large as compared to the lot size since $n_c < N < n_{c+1}$.

Since log N is nearly a linear function of c, at least for large lots, rather accurate results may be obtained by corresponding interpolation. For applications in practice it is, however, hardly worth while using logarithms, linear interpolation in N will normally suffice.

To find a sampling plan for a lot size not used as argument in the table the first step should thus be to <u>determine c by linear interpolation with respect to N</u> and round the result to the nearest integer. It should then be noted that <u>n is a function of c and p_2 only</u>, i.e. n is independent of p_1, so that n may in many cases be found corresponding to the given c in another column of the same LTPD table. If that is not so the nearest neighbouring values to the given c may be

found and n may be determined by linear interpolation with respect to c. Another possibility is to use the formula given in section 9.

As an example consider the problem of determining the sampling plan for $N = 1600$, $LTPD = 5\%$, $p_1 = 2.5\%$, and $\gamma = 1$. Linear interpolation gives $c = 12$ and looking for n corresponding to $c = 12$ in another column of the same LTPD table it will be seen that $n = 353$. Changing N to 16,000 linear interpolation gives $c = 27$. The nearest values of c in the table are 25 and 28 with the corresponding n-values of 651 and 718. Linear interpolation gives $n = 696$. Sometimes n may be found directly in the corresponding LTPD table for $\gamma = 5$.

Numerical investigations have shown that the proposed method of interpolation will ordinarily give the correct value of c but may result in an error of one unit. As pointed out previously it is essential to use the right method to determine n when c has been found to secure that $P(p_2) = 0.10$. If the rules stated are followed the plans determined by interpolation will be optimum or very nearly so since the minimum of the cost function is rather broad.

It is customary in practice to set up rather large intervals for N and use the same sampling plan for all N within an interval. The present tables may easily be used for constructing such intervals in two ways:

(1) The tabular values of N may be considered as "midpoints" of the following intervals:

N	Interval
100	85 - 150
200	150 - 250
300	250 - 400
500	400 - 600
700	600 - 850

(2) The tabular values of N may be considered as upper endpoints of such intervals which means that too large sample sizes will be used in all cases.

Whatever procedure is applied for constructing such intervals the result will be that the sampling plan used for a certain interval will only be optimum for that part of the interval which is given by (N_{c-1}, N_c) where c is the acceptance number used. For all other parts of the interval the costs will be larger than necessary.

In a previous paper [7] similar tables have been given based on a hypergeometric consumer's risk and a binomial producer's risk as in the Dodge-Romig tables, and the relations between the solutions in the three cases (Poisson, Binomial, Hypergeometric) have been discussed. A comparison of the present tables and

the previous ones shows that in most cases a hypergeometric and a binomial consumer's risk of 10 per cent will lead to the same value of c or values of c differing only by 1. Only for $p_2 > 0.05$ and $r > 0.5$ do the tables contain values of c differing by 2 and occasionally 3 and 4 units. One may therefore conclude that the values of c found in the present tables may also be used for the case with a hypergeometric consumer's risk. The corresponding sample size, n_h say, may be determined from the binomial n with good approximation from the formula

$$n_h = n\,\{1 - (np_2 - c)/2Np_2)\}. \tag{45}$$

As an example consider the problem of determining the optimum sampling plan with a 10 per cent hypergeometric consumer's risk for $N = 200$, LTPD = 5%, $p_1 = 1\%$, and $\gamma = 1$. The present tables show that the "binomial solution" is $n = 77$ and $c = 1$. From (45) we find $n_h = 77 \times 0.857 = 66$ which actually is the correct result.

In [6] has been given a discussion of how to use the Poisson solution to obtain an approximation to both the binomial and the hypergeometric solution. One of the advantages of the present Poisson tables is that they contain the solution for 14 values of r whereas the other tables only have 5 values of r. The Poisson tables are therefore useful when sampling plans are needed for values of p_1 or p_2 not contained in the other tables.

The plans have been tabulated for two values of γ only, γ = . and $\gamma = 5$. Plans for other values of γ may be obtained from these tables by using the result of section 5 that the optimum sampling plan asymptotically only depends on the product of lot size and cost parameter. This leads to the following two rules:

(1) For $\gamma \leq 3$ and a given N compute $N^* = N\gamma$ and use the plan corresponding to N^* in the table for $\gamma = 1$.

(2) For $3 < \gamma < 10$ and a given N compute $N^* = N\gamma/5$ and use the plan corresponding to N^* in the table for $\gamma = 5$.

Numerical investigations have shown that the two rules give remarkably good approximations to the optimum sampling plans also for small values of N which means that practically all cases for $\gamma < 10$ have been covered by means of the two given tables. The table for $\gamma = 1$ tends to give too low an acceptance number when used for $\gamma < 1$ and too large an acceptance number when used for $\gamma > 1$ and analogous results hold for $\gamma = 5$. In most cases, however, the correct acceptance number will be found or the error will be at most one unit. It should also be noted that the error tends to increase with r.

It follows that the largest deviations from the exact values of c for $\gamma < 5$ may be expected to occur for values of γ around 3. To demonstrate how the formulas work in the worst case an example has been given in Table 1 where the acceptance numbers for $\gamma = 3$ have been derived from both tables. It will be

seen that the values of c found deviate at most 1 from the correct values apart from one case where the deviation is 2.

Table 1.

LTPD plans with minimum costs for $100p_2 = 5$ and $100p_1 = 2$.

Values of c for $\gamma = 3$ computed from $\gamma = 1$ and $\gamma = 5$ compared to the exact values of c.

N	Exact c	$N^* = 3N$	c	$N^* = 0.6N$	c
30	All	90	0	18	All
50	0	150	0	30	All
70	0	210	1	42	0
100	1	300	2	60	0
200	4	600	4	120	2
300	5	900	6	180	4
500	7	1500	8	300	7
700	9	2100	10	420	8
1000	11	3000	12	600	10
2000	14	6000	14	1200	13
3000	16	9000	16	1800	15
5000	18	15000	17	3000	17
7000	19	21000	19	4200	18
10000	20	30000	20	6000	20
20000	23	60000	23	12000	23
30000	24	90000	25	18000	24
50000	26	150000	26	30000	26
70000	28	210000	28	42000	28
100000	29	-	-	60000	29
200000	32	-	-	120000	32

Denoting the upper limit for $M = Np_2$ by $M(c,\gamma)$ we have the following approximate relations for the Poisson tables: For $\gamma \leq 3$ use $M(c,\gamma) = M(c,1)/\gamma$ and for $3 < \gamma < 10$ use $M(c,\gamma) = M(c,5)5/\gamma$, i.e. compare $M^* = M\gamma$ and $M^* = M\gamma/5$ with the limits given in the two tables.

Example 1. Suppose that a producer inspects lots of 1,000 items each and that he has decided on a LTPD of 5%. His average quality under normal conditions is supposed to be 1% defectives. It is furthermore assumed that $k_a(p) = 0$ from the producer's point of view, that rejection means sorting, and that costs of sorting are the same as costs of sampling inspection per item. According to (42) these assumptions lead to $\gamma = 1$ and the corresponding optimum plan may therefore be found directly in the table as n = 132 and c = 3. If, however, sorting costs are only half of sampling inspection costs per item, i.e. $\gamma = 0.5$, then the same table should be used with $N^* = 0.5N = 500$ which gives the optimum plan n = 105 and c = 2.

If rejection means rework of the whole lot and the costs of rework per item equals the double of sampling inspection costs, i.e. $\gamma = 2$, then the table should be entered with $N^* = 2N = 2{,}000$ which gives the plan $n = 158$ and $c = 4$. Had γ been 4 instead of 2 then the table for $\gamma = 5$ should be entered with $N^* = 4N/5 = 800$ which leads to $n = 184$ and $c = 5$.

Suppose now that a prior distribution of p gives probability $w_1 = 0.85$ to $p = 0.01$ and probability $w_2 = 0.15$ to $p = 0.05$, that the assumptions about the costs are as above, and that the producer wants to minimize average costs under the restriction $P(0.05) = 0.10$. From (44) we then find $\gamma = 1/1.0176 = 0.98$ as compared to $\gamma = 1$ above. Therefore we find the same sampling plan. For the other three cases we find in the same manner $\gamma = 0.46$, 2.33, and 7.39, respectively. The only important change is from 4 to 7.39 which may lead to change the sampling plan (184,5) to (209,6).

Example 2. In [8] an example with $N = 500$, $w_1 = 0.93$, $p_1 = 0.009$, $w_2 = 0.07$, $p_2 = 0.080$, $\gamma_1 = 0.567$, and $\gamma_2 = 0.168$ has been discussed in details and it has been shown that the Bayesian single sampling plan is $n = 30$ and $c = 1$. This plan, however, gives a consumer's risk of 29.6% which in certain cases may be considered unsatisfactory, and we shall therefore find the restricted Bayes solution with a consumer's risk of 10%.

As p_1 and p_2 are rather small and are not to be found in the tables with binomial probabilities we shall first derive the solution by means of the Poisson tables. Since $\gamma = 0.567/(1 - 0.0168) = 0.577$ and $M = 500 \times 0.080 = 40$ we find $M\gamma = 23.1$. From the table for $\gamma = 1$ and $r = 0.11$ we read $c = 1$ and $n = 3.889/0.080 = 48.6$ which gives the binomial $n_b = 47$ using the formula $n_b = n - (np_2 - c)/2$, see [6]. From a table of the binomial distribution we find $P(p_2) = 0.10104$ for $n = 47$ and $P(p_2) = 0.09455$ for $n = 48$. Using $n = 48$ and $c = 1$ (so that $P(p_2) \leq 0.10$) we find $Q(p_1) = 0.06961$ and finally $R = 48 + 25.0 = 73.0$. For the Bayes solution the corresponding results are $Q(p_1) = 0.0298$ and $P(p_2) = 0.2958$ giving $R = 30 + 31.3 = 61.3$. The price to be paid for the restriction required may thus be expressed by means of the increase in costs from 61.3 to 73.0.

7. AQL sampling inspection plans with minimum costs.

AQL plans are here defined as sampling plans with a given Acceptable Quality Level, $100p_1$, and a corresponding probability of acceptance, $P(p_1)$, which traditionally is chosen as 95 per cent.

An analysis similar to the one in the previous section may be carried out from the point of view of a consumer inspecting submitted lots. Suppose that the consumer

has the following two main objectives: (1) To make reasonably sure that lots of satisfactory quality are accepted. (2) To keep his inspection costs and decision losses down.

One may now proceed formally as in section 6, i.e. select an upper limit, p_1, for the acceptable process average and a corresponding risk for the producer, $Q(p_1)=0.05$ say, and then minimize the consumer's average costs for lots of unsatisfactory quality, $K(N,n,c,p_2)$, under this restriction. This procedure is, however, not satisfactory since it corresponds to a restricted Bayes solution with a one-point distribution giving probability 1 to $p = p_2$, i.e. the consumer's costs are minimized under the assumption that all submitted lots are unsatisfactory, and this will naturally give too large samples.

We shall therefore analyse the problem under the assumption that the prior distribution of p is a two-point distribution with parameters (p_1, p_2, w_2).

If the parameters are known and the distribution is stable and if $p_1 < p_r < p_2$ the Bayes solution may be determined as described in [8]. If, however, the stability of the prior distribution is doubtful and/or information on costs is incomplete the consumer may prefer a restricted Bayes solution.

Firstly the consumer may find it necessary to protect himself against some of the consequences of a deterioration of the prior distribution and for that reason he may impose the condition $Q(p_1) = 0.05$ on the plans. This should also induce the producer to keep the main component of the prior distribution at the level p_1 or lower.

Secondly the costs of rejection may be (partly) unknown to the consumer because even if they may seem small in the short run rejection of good lots may in the long run involve higher prices, difficulties in getting contracts, delayed deliveries, etc.

The consumer may therefore be forced to modify the model. Instead of minimizing the complete cost function without any restrictions he may choose to minimize the incomplete cost function obtained by putting $k_r(p) = 0$ under the restriction $Q(p_1) = 0.05$ hoping that the resulting low frequency of good lots rejected will reduce his costs of rejection sufficiently.

One of the effects of fixing $Q(p_1)$ may be judged by noting that on the average the ratio of number of good lots rejected to total number of lots rejected will be $w_1Q(p_1)/(w_1Q(p_1) + w_2Q(p_2))$. For $w_2 = 0.10, Q(p_1) = 0.05$, and $Q(p_2) \approx 1$, say, this ratio will be about 1/3.

The sampling plan is determined by minimizing the average costs, $K(N,n,c)$, under the restriction of a fixed producer's risk, $Q(p_1) = 0.05$. This is equivalent to

minimizing $R_o = n + (N-n)\gamma P(p_2)$ with

$$\gamma = \frac{w_2(k_a(p_2) - k_r(p_2))}{w_2(k_s(p_2) - k_r(p_2)) + w_1(k_s(p_1) - 0.95k_a(p_1) - 0.05k_r(p_1))} = \frac{\gamma_2}{1 - 0.05\gamma_1} \quad (46)$$

which problem has been solved in section 4.

Let us consider the case where $k_r(p) = 0$. If further $k_s(p) = S_1$ and $k_a(p) = A_2p$ we may introduce the break-even quality $p_s = S_1/A_2$, i.e. the ratio between sampling inspection costs per item of the sample and the costs resulting from accepting a defective item. From (46) we then have

$$\gamma = w_2p_2/(p_s - 0.95w_1p_1) \quad (47)$$

which is an increasing function of w_2 taking on the maximum value p_2/p_s for $w_2 = 1$.

In general, if w_1 and p_1 are known only approximately, w_1 may be chosen as small as reasonable and p_1 as large as reasonable to find an upper limit for γ and a correspondingly large sample size.

The tables based on Poisson probabilities give $m = np_1$ and $M = Np_1$ as functions of c so that the optimum plan is (c,m) for $M_{c-1} < M < M_c$. The two functions have been tabulated for $r = p_2/p_1 = 1.50$, 1.60, 1.80, 2.00, 2.25, 2.50, 2.75, 3.0, 3.5, 4.0, 5.0, 6.5, 10.0, for $\gamma = 0.2$ and 1.0, and for $c \leq 99$ with the modification that tabulation has been stopped when M exceeds 50,000. Because only an abridged version is published the last figure for M given in a column may be less than 50,000 even if $c < 99$ which means that M exceeds 50,000 for the next entry.

The tables based on binomial probabilities give (n,c) as functions of N for 20 values of N between 30 and 200,000. Plans have been computed only for $c \leq 99$. As values of the parameters have been used $100p_1 = 0.1$, 0.2, 0.5, 1, 2, 3, 4, 5, 7, 10, five values of $r = p_2/p_1$ chosen among the values 1.5, 1.7, 2.0, 2.5, 3.0, 4.0, 5.0, 6.0, 10.0 (with small modifications), and $\gamma = 0.2$ and 1.0, giving a total of 10 x 5 x 2=100 tables. The tables also contain $P(p_2)$ which makes it easy to compute R_o, $R = (1 - 0.05\gamma_1)R_o + 0.05N\gamma_1$, and the average costs $K = (k_s - k_m)R + Nk_m$.

For $\gamma < 1$ it may happen that acceptance without inspection is cheaper than sampling inspection for small lots. In such cases the cheapest sampling plan available (c as small as possible) has nevertheless been tabulated and the letter a (for acceptance) has been added after the sample size.

The same methods of interpolation as described for the LTPD plans should be used here.

For $\gamma \leq 0.6$ plans may be found from $N^* = N\gamma/0.2$ and the tables for $\gamma = 0.2$, whereas for $0.6 < \gamma < 2.0$ the tables for $\gamma = 1.0$ should be used with $N^* = N\gamma$.

"Interval-tables" for N may be constructed as indicated for the LTPD tables.

Applications of the AQL system of sampling plans with fully specified prior distribution and cost parameters do not cause any difficulties, see Examples 3 and 4.

A comparison of the present system with other AQL systems is rather difficult since the other systems only partly are based on explicity formulated mathematical assumptions. The systems chosen for comparison are the SRG system [12], the SMS (Swedish Military Standard) system [10], and the system recently proposed by Dodge [4]. The Military Standard 105 has not been included because the acceptance probability at the AQL value is not constant but an increasing function of lot size.

To carry out such a comparison it is obviously necessary to simplify the present system in particular with respect to the cost parameters because the other systems do not have explicitly formulated assumptions regarding costs. Using the assumptions leading to (47), and assuming furthermore (arbitrarily) for the break-even quality that $p_s = \sqrt{p_1 p_2}$ we find for small values of w_2

$$\gamma \approx w_2 p_2/(p_s - p_1) = w_2 r/(\sqrt{r}-1), \quad r = p_2/p_1.$$

The function $r/(\sqrt{r}-1)$ attains its minimum which is equal to 4 for $r = 4$ and does not exceed 5 for $2 \leq r \leq 13$ which is the domain of interest in practice. Under the assumptions stated we may therefore use $4w_2$ as a rough approximation to γ.

Table 2 contains comparisons of acceptance numbers for 5 AQL values and 7 lot sizes (Sample size is the same function of acceptance number for the four systems). For the three other systems the recommended normal inspection level has been used. The present system has been denoted RB (Restricted Bayes) and the parameters have been chosen as $r = 4$ and $w_2 = 0.1$ giving $\gamma = 0.4$. The acceptance numbers have been found by entering the Poisson table for $\gamma = 0.2$ with $N^* = 2Np_1$.

Table 2.

Comparison of acceptance numbers.

AQL	0.1%				0.4%			1.0%				4.0%				10.0%		
N	SRG	D	SMS	RB	SRG	D	RB	SRG	D	SMS	RB	SRG	D	SMS	RB	D	SMS	RB
100	0	All	All	0	1	All	0	1	1	2	0	3	2	2	1	3	4	3
300	0	All	All	0	1	1	0	1	1	2	1	4	3	2	3	7	4	5
1000	1	1	1	0	2	1	1	3	2	3	3	8	7	5	5	15	8	6
3000	1	1	1	1	2	2	3	4	3	3	5	10	10	6	6	22	10	8
10000	1	1	2	3	4	2	5	7	5	5	6	18	15	9	8	22	12	9
30000	1	1	3	5	5	3	6	9	7	6	8	26	22	9	9	22	15	10
100000	1	1	4	6	5	5	8	9	10	8	9	26	22	11	10	22	15	11

It will be seen that the SMS and the present system give nearly the same results whereas the SRG and Dodge's systems have smaller acceptance numbers for small AQL's and larger for large AQL's.(The same is true for the SMS but to a much smaller degree). One way of obtaining similar results as the SRG and Dodge within the present framework is to make r a function of p_1. From a practical point of view it seems a reasonable explanation that the SRG and Dodge have implied that the ratio between the typical bad and good quality level decreases with increasing values of the quality level itself. Looking for the values of r which will give the acceptance numbers in Table 2 we find approximately the following results:

	SRG and D		SMS	
$100p_1$	r	$100p_2$	r	$100p_2$
0.1	10.0	1.0	6.5	0.65
0.4	6.5	2.6	-	-
1.0	4.0	4.0	4.0	4.0
4.0	2.5	10.0	4.0	16.0
10.0	2.0	20.0	3.5	35.0

The same idea has actually been built into the tables based on binomial probabilities since the solution has been tabulated for values of r between 2 and 10 for $100p_1 = 0.1$ decreasing to values of r between 1.5 and 3 for $100p_1 = 10.0$.

It thus seems that the other systems have a simple interpretation within the present model. The arbitrary relationship between lot size and sample size in these systems may be converted to an (arbitrary) relationship between p_1 and p_2 which, however, is easier to interpret and understand. It will normally be much easier to reach a motivated decision with respect to the choice of p_2 than with respect to "inspection level".

Another way of influencing the amount of inspection is by varying γ, i.e. w_2, which has the simple effect of changing the "effective lot size". If w_2 is changed from 0.10 to 0.05 the same table should be entered with a lot size half the original one.

The other systems may possibly be obtained from the present one in various other ways, but the above model seems to give one of the simplest and most useful interpretations containing only two (p_2 and w_2) adjustable parameters. (The other systems possess a number of simple properties valuable from an administrative point of view which, however, have not been included in the discussion above).

Example 3. Suppose that a consumer inspects lots of 3,000 items each coming from a process with probability $w_1 = 0.85$ for $p = 0.01$ and probability $w_2 = 0.15$ for $p = 0.03$. It is furthermore assumed that $k_r(p) = 0$ from the consumer point of view, that sampling inspection costs are 0.15 units per item, and that the costs of accepting a defective are 10.0 units, which gives a break-even quality of $p_s = 0.015$. According to (47) these assumptions lead to $\gamma = 0.650$. The sampling plan may therefore be found

in the table for $\gamma = 1$ with $N^* = 3000 \times 0.650 = 1950$ which gives $n = 399$ and $c = 7$. The same result may be obtained from the table for $\gamma = 0.2$ with $N^* = 3000 \times 0.650/0.2 = 9750$.

<u>Example 4.</u> Using the data from Example 2, but changing the condition from $P(p_2) = 0.10$ to $Q(p_1) = 0.05$, we find $\gamma = 0.168/(1-0.05 \times 0.567) = 0.173$. From $M = 500 \times 0.009 = 4.5$ we find $M^* = 4.5 \times 0.173/0.2 = 3.89$ which should be compared to M_c in the Poisson table for $\gamma = 0.2$ and $r = 0.080/0.009 = 8.9$. The result is $c = 1$ and $n = 0.3555/0.009 = 39.5$. A table of the binomial distribution shows that $Q(p_1) = 0.04818$ for $n = 39$ and $Q(p_1) = 0.05042$ for $n = 40$ so that $n = 39$ must be preferred if the condition $Q(p_1) \leq 0.05$ has to be respected. As $P(p_2) = 0.16995$ we find $R = 39 + 25.8 = 64.8$ which exceeds the Bayesian costs by 3.5.

<u>8. IQL sampling inspection plans with minimum costs.</u>

IQL plans are here defined as <u>sampling plans with a given Indifference Quality Level,</u> $100p_o$, <u>and a corresponding probability of acceptance</u> $P(p_o) = 1/2$.

The IQL plans are particularly well suited for use in cases where the producer and the consumer are parts of the same firm. The reasons for using the restriction $P(p_o) = 1/2$ are of a similar nature as those discussed in the two previous sections.

It is clear that all the results regarding LTPD plans with a one-point distribution of p may be used analogously for IQL plans based on minimization of $R_o = n + (N-n)\gamma Q(p_1)$. Such plans are generalizations of the plans discussed by Weibull [13] and tabulated by Markbäck [11] in the same sense as the LTPD plans are generalizations of the Dodge-Romig plans.

For a two-point prior distribution, however, new problems arise since it is not possible to reduce $R = n + (N-n)(\gamma_1 Q(p_1) + \gamma_2 P(p_2))$ as for the LTPD and AQL plans because the restriction $P(p_o) = 1/2$, $p_1 < p_o < p_2$, cannot be used to "eliminate" $Q(p_1)$ or $P(p_2)$ from R. The restriction leads to the desirable result that <u>both the producer's and the consumer's risks tend to zero with increasing sample size.</u>

The restriction

$$P(p_o) = B(c,n,p_o) = 1/2 \tag{48}$$

defines a relation $n = n_c$ between n and c. Proceeding as in section 4 we find that the plan (c,n_c) is optimum for $N_{c-1} < N < N_c$ where

$$N_c = n_{c+1} - (1-\gamma_1 Q(p_1) - \gamma_2 P(p_2))\Delta n_c/(\gamma_1 \Delta Q(p_1) + \gamma_2 \Delta P(p_2)),$$

$Q(p_1) = 1-B(c,n_c,p_1)$, and $P(p_2) = B(c,n_c,p_2)$. Optimum plans may therefore be tabulated by a similar procedure as described in section 4 the only essential difference being

that the plans here depend on five parameters $(p_o, p_1, p_2, \gamma_1, \gamma_2)$ instead of three (p_1, p_2, γ).

The asymptotic properties of the system may be found as in section 5. The condition (48) corresponds to

$$c = np_o - \frac{1}{3}(2 - p_o) + O(1/n), \tag{49}$$

see (26), or

$$\frac{c}{n} = h = p_o + \frac{b}{n} + O(1/n^2), \tag{50}$$

where $b = -(2 - p_o)/3$.

Setting

$$f(n, p_i) = \frac{\lambda_i}{\sqrt{n}} e^{-n\varphi(h, p_i)} \tag{51}$$

and

$$\lambda_i = \frac{q_o}{\sqrt{2\pi p_o q_o}} \quad \frac{\gamma_i p_i}{|p_o - p_i|} \tag{52}$$

we find by means of formulas analogous to (29) and (34)

$$R = n + (N-n)(f(n,p_1) + f(n,p_2))(1 + O(n^{-\frac{1}{2}}))$$

and

$$\varphi(h, p_i) = \varphi(p_o, p_i) + \frac{b}{n} \ln \frac{p_o q_i}{q_o p_i} + O(n^{-\frac{3}{2}}). \tag{53}$$

For $n \to \infty$ one of the exponential terms will be infinitely small as compared to the other depending on which of the two coefficients $\varphi(p_o, p_1)$ and $\varphi(p_o, p_2)$ is the larger. The solution will therefore have the same asymptotic properties as those previously studied with the exception that instead of having one risk fixed and the other inversely proportional to N, both risks will here tend to zero, one inversely proportional to N and the other inversely proportional to $N^{\varphi(p_o, p_2)/\varphi(p_o, p_1)}$, say, if $\varphi(p_o, p_2) > \varphi(p_o, p_1)$.

This lack of symmetry will normally be considered unreasonable, and unless there exist strong reasons to the contrary p_o should be chosen so that $\varphi(p_o, p_1) = \varphi(p_o, p_2)$ which gives

$$p_o = \left(\log \frac{q_1}{q_2}\right) \Big/ \left(\log \frac{q_1 p_2}{q_2 p_1}\right). \tag{54}$$

For this value of p_o we find

$$R = n + (N-n) \frac{\lambda_o}{\sqrt{n}} e^{-n\varphi_o} (1 + O(n^{-\frac{1}{2}})) \tag{55}$$

where $\varphi_o = \varphi(p_o, p_1) = \varphi(p_o, p_2)$ and

$$\lambda_o = \lambda_1 e^{-b\varphi_1'} + \lambda_2 e^{-b\varphi_2'}, \quad \varphi_i' = \ln(p_o q_i / q_o p_i). \tag{56}$$

Minimization with respect to n gives

$$\ln(N-n) = \varphi_o n + \frac{1}{2}\ln n - \ln(\lambda_o\varphi_o) + o(1) \tag{57}$$

and

$$\min R = n + 1/\varphi_o + o(1). \tag{58}$$

For the risks we have

$$Q(p_1) = \frac{\lambda_1 e^{-b\varphi_1'}}{\lambda_o\varphi_o\gamma_1}\ \frac{1}{N-n} + o(\frac{1}{N}), \tag{59}$$

and a similar expression for $P(p_2)$ which means that

$$P(p_2)/Q(p_1) \to \frac{p_2(p_o - p_1)}{p_1(p_2 - p_o)}\ e^{b(\varphi_1' - \varphi_2')} = \rho, \tag{60}$$

say. Since $b \simeq -2/3$ we find

$$\rho \simeq \frac{p_o - p_1}{p_2 - p_o}\left(\frac{p_2}{p_1}\right)^{1/3}\left(\frac{q_2}{q_1}\right)^{2/3} . \tag{61}$$

To find an approximation to ρ for small values of (p_1, p_2) we let $p_1 \to 0$ for fixed $r = p_2/p_1$ which leads to

$$\rho \to \frac{r - 1 - \ln r}{r\ln r - r + 1}\, r^{1/3}. \tag{62}$$

The limiting value of ρ increases slowly from 1.00 to 1.06 for r increasing from 1 to 20. For most purposes it will be sufficiently accurate to use $\rho = 1$.

By means of (49) and (57) we may find good approximations to the IQL plans.

There exists, however, another possibility of approximating these plans by making use of the property (60). Writing

$$R = n + (N-n)Q(p_1)(\gamma_1 + \gamma_2 P(p_2)/Q(p_1))$$

and noting that (60) does not depend on the minimization but only on the restriction $P(p_o) = 1/2$ we have for large N that

$$R \sim n + (N-n)(\gamma_1 + \rho\gamma_2)Q(p_1). \tag{63}$$

It seems therefore reasonable to use the IQL plan defined by the parameters (p_o, p_1, γ), $\gamma = \gamma_1 + \rho\gamma_2$, i.e. a plan based on a one-point distribution of p, as approximation to the IQL plan defined by $(p_1, p_2, \gamma_1, \gamma_2)$.

The above result may also be derived formally from the asymptotic expressions for R. Consider the problem of minimizing $R_o = n + (N-n)\gamma Q(p_1)$ for an arbitrary γ under the

restriction $P(p_o) = 1/2$. Proceeding as in section 5 we find

$$R_o = n + (N-n)\frac{\lambda_1\gamma}{\gamma_1}e^{-b\varphi_1'}\frac{1}{\sqrt{n}}e^{-n\varphi_o}(1 + O(n^{-\frac{1}{2}})), \qquad (64)$$

which is analogous to (53). Comparing (64) and (55) it will be seen that the two expressions are identical for

$$\frac{\lambda_1\gamma}{\gamma_1}e^{-b\varphi_1'} = \lambda_o$$

and solving for γ we find $\gamma = \gamma_1 + \rho\gamma_2$.

It should furthermore be noticed that the IQL plans and the Bayesian plans defined by $(p_1, p_2, \gamma_1, \gamma_2)$ have the same asymptotic properties, see [8]. For the Bayesian plans we have that $c = np_o + a_o + o(1)$ which means that asymptotically $P(p_o) = 1/2$. The asymptotic form of R which has to be minimized with respect to n to find the Bayesian plan is identical to (55) with a_o substituted for b. The essential difference between the Bayesian sampling plan and the corresponding IQL plan lies therefore in the different constant terms of the linear relations between c and n. A good approximation to the Bayesian plan may therefore be found be looking up the value of c in the table of the corresponding IQL plan and computing n from c by means of the correct (Bayesian) relation, see the examples later in this section.

It follows that the IQL plans have economic efficiency 1 for $N \to \infty$ as compared to the Bayesian plans.

The IQL plans have thus very desirable properties:

(1) The restriction $P(p_o) = 1/2$ corresponds practically to a linear relation between n and c.

(2) The relation between n and N is approximately equal to $\ln(N-n) = \varphi_o n + \frac{1}{2}\ln n - \ln(\lambda_o\varphi_o)$.

(3) The producer's and consumer's risks are nearly equal and tend to zero inversely proportional to N.

(4) Asymptotically the minimum costs are $n + 1/\varphi_o$, i.e. decision losses tend to zero as compared to sampling inspection costs.

(5) The plans for a double binomial prior distribution may be found approximately from the plans for a single binomial distribution which reduces the necessary tables greatly.

(6) The IQL plans have asymptotic efficiency equal to 1 as compared to the Bayesian plans.

(7) The IQL plans may be used to find good approximations to the

The tables of IQL plans are based on a one-point prior distribution and correspond to the previously discussed tables of LTPD plans. The rules given in section 6 for interpolation, construction of "interval-tables", and change of cost parameter may therefore be applied.

The tables based on Poisson probabilities give $M(c,\gamma)$ for $\gamma = 1$ and $c \leq 99$ for $r = p_1/p_o = 0.10, 0.15, \ldots, 0.80$.

The tables based on binomial probabilities show the optimum plans for $100p_o = 0.5$, 1,2,3,4,5,7,10,15, for five values of r chosen among the values 0.2,0.3,...,0.8, and for $\gamma = 1$, giving a total of 45 tables.

Tables are given for $\gamma = 1$ only since these may be used to find plans for all $\gamma < 10$ by intering the tables with $N^* = N\gamma$.

On the basis of the asymptotic theory above it has been postulated that the tabulated IQL plans which are based on a one-point prior distribution may be used to find the IQL plans for a two-point prior distribution with good approximation also for small values of N. This has been confirmed by numerical investigations, and a few typical examples based on Poisson probabilities are shown below for $k_r(p) = k_s(p)$, $p_r = 0.01$, and $w_2 = 0.05$.

Table 3.

Comparisons of acceptance numbers for equivalent IQL plans based on one- and two-point prior distributions.

N	$p_1 = 0.004$ $p_2 = 0.060$ $\gamma_2 = 0.439$	$p_o = 0.02$ $\rho = 0.949$ $\gamma = 1.417$	$p_1 = 0.0050$ $p_2 = 0.0175$ $\gamma_2 = 0.0790$	$p_o = 0.010$ $\rho = 1.004$ $\gamma = 1.079$	$p_1 = 0.006$ $p_2 = 0.015$ $\gamma_2 = 0.066$	$p_o = 0.010$ $\rho = 1.079$ $\gamma = 1.071$
300	0	0	0	0	0	0
500	1	1	0	0	0	0
700	1	1	0	0	0	0
1000	1	1	1	1	1	1
2000	2	2	2	2	2	2
3000	2	2	3	3	3	3
5000	3	3	5	5	6	6
7000	3	3	6	6	7	7
10000	4	4	7	7		

Under the assumptions stated $\gamma_1 = 1$ and p_o, ρ, and γ have been computed from (54), (61), and $\gamma = \gamma_1 + \rho\gamma_2$. It will be seen that the approximation is excellent also for small values of N even in cases where p_2/p_1 is quite small.

It has also been postulated that the acceptance number for the Bayesian sampling plan based on a two-point distribution is approximately equal to the acceptance number for the "equivalent" IQL plan for a one-point

distribution. Numerical investigations have confirmed that the approximation is good for $p_2/p_1 > 5$, whereas deviations of 1 or 2 may occur for $3 < p_2/p_1 < 5$. For $p_2/p_1 < 3$ the approximation is usually poor. The following typical examples are based on the same assumptions as in Table 3.

Table 4.

Comparisons of acceptance numbers for Bayesian plans and equivalent IQL plans.

N	p_1=0.004 p_o=0.02, p_2=0.060 ρ=0.949, γ_2=0.439 γ=1.416		p_1=0.004 p_o=0.010, p_2=0.020 ρ=1.015, γ_2=0.088 γ=1.089		p_1=0.0050 p_o=0.010, p_2=0.0175 ρ=1.004, γ_2=0.0790 γ=1.079		p_1=0.006 p_o=0.010, p_2=0.015 ρ=1.079, γ_2=0.066 γ=1.071	
300	1	0	Accept	0	Accept	0	Accept	0
500	1	1	"	0	"	0	"	0
700	1	1	"	0	"	0	"	0
1000	2	1	"	1	"	1	"	0
2000	2	2	"	2	"	2	"	2
3000	3	2	"	2	"	3	"	3
5000	3	3	3	3	"	4	"	5
7000	3	3	3	4	"	6	"	7
10000	4	3	4	5	5	7	"	9
30000	5	5	7	8	10	11	11	16
100000	6	6	11	11	15	17	21	25

Example 5. The data are as in Example 1 with the modification that the producer has decided on an IQL of 3% instead of a LTPD of 5%. For $p_1 = 0.01$, $p_o = 0.03$, $\gamma = 1$, and $N = 1000$ we find the IQL plan in the table as $n = 89$ and $c = 2$. If γ had been equal to 1/2 instead of 1 the table should have been entered with $N^* = 500$ which gives the plan $n = 56$ and $c = 1$.

Suppose now that there exists a prior distribution of p with probability $w_1 = 0.85$ for $p = 0.01$ and probability $w_2 = 0.15$ for $p = 0.05$ and that the producer wants to minimize average costs under the restriction $P(p_o) = 1/2$ where p_o is determined by (54), i.e. $p_o = 0.0250$. For $k_r(p) = k_s(p)$ and $k_a(p) = 0$ we find $\gamma_1 = 1$ and $\gamma_2 = -0.176$. From (61) it follows that $\rho = 0.998$ so that $\gamma = \gamma_1 + \rho\gamma_2 = 0.824$. For $N^* = 824$ the tables for IQL = 2% and 3% give $c = 1$ and 2 respectively. Consulting the Poisson table for $r = p_1/p_o = 0.40$ and $M^* = 25 \times 0.824 = 20.6$ it will be seen that $c = 2$ is to be preferred. From (49) we then find $n = 106$.

Example 6. To find the IQL plan for the data in Example 2 we first compute $p_o = 0.033$ by means of (54), $\rho = 1.001$ from (61), and $\gamma = \gamma_1 + \rho\gamma_2 = 0.735$. Entering the IQL Poisson table with $M^* = 500 \times 0.033 \times 0.735 = 12.1$ and $r = 0.009/0.033 = 0.27$ we find $c = 1$ and $n = 1.678/0.033 = 51$. The binomial probability $P(p_o) = 0.49496$. From $Q(p_1) = 0.07732$ and $P(p_2) = 0.07733$ we find $R = 51 + 25.5 = 76.5$.

Using $c = 1$ to find the Bayesian n_c we compute $\alpha = (\log \frac{\gamma_2}{\gamma_1})/(\log \frac{q_1}{q_2}) = -16.4$,

$\beta = 1/p_o = 30.3$, and $n_c = -16.4 + 30.3 \times 1.5 = 29$ as compared to the exact solution $n = 30$.

The results found in Examples 2, 4, and 6 have been summarized in the following table.

Plan	c	n	R	100Q(p_1)	100P(p_2)
LTPD	1	48	73.0	7.0	9.5
AQL	1	39	64.8	4.8	17.0
IQL	1	51	76.5	7.7	7.7
Bayes	1	30	61.3	3.0	29.6

9. The OC-curve.

Let the solution of the equation $100P(p) = \alpha$, $0 < \alpha < 100$, be denoted by p_α. From (49) we have with good approximation

$$p_{50} = (c + \tfrac{2}{3})/(n + \tfrac{1}{3}),$$

so that p_{50} may be easily found for any given sampling plan.

In [6] it has been shown that an approximate solution to the equation $100B(c,n,p) = \alpha$ may be found by first solving the corresponding Poisson equation $100B(c,m) = \alpha$ with respect to m and then computing

$$p = m/(n + \tfrac{m-c}{2}).$$

The accuracy of this approximation has been checked numerically for p_{10} and p_{95}. The relative error is normally a decreasing function of c and an increasing function of p.

For p_{10} the relative error is less than 0.5% for all c and for all $p < 0.20$ which means that the formula gives p_{10} to three significant figures in practically all cases.

For p_{95} the relative error is less than 0.5% for all c and all $p < 0.05$, less than 1% for $p = 0.10$, and less than 2% for $p = 0.20$. (For $p = 0.20$ the statement does not hold for $c = 1$ where the relative error is 4%).

Values of m as function of c may be found in the LTPD and AQL Poisson tables in the Appendix, or in a table of χ^2-fractiles.

By using this simple formula and the information given in the sampling tables it follows that at least four points on the OC-curves are known or easily found.

Consider for example the LTPD plan for $p_2 = 0.10$, $p_1 = 0.04$, $\gamma = 1$, and $N = 300$, which give $(n,c) = (78,4)$. The table gives $P(p_1) = 0.798$ and $P(p_2) = 0.10$. The formulas above give $p_{50} = 0.060$ and $p_{95} = 1.970/(78-1.02) = 0.0256$.

It should also be noted that the same plan may occur in other columns of the same

LTPD table or in the corresponding table for $\gamma = 5$ in which case further values of $100P(p)$ may be read from the table. In the example above we find in the table for $\gamma = 5$ and $p_1 = 0.03$ the result $P(p_1) = 0.915$.

Consider the plan for $N = 500$ instead of $N = 300$. The plan is $(n,c) = (91,5)$ and it occurs in all columns of the table either for $\gamma = 1$ or $\gamma = 5$. Therefore 6 points on the OC-curve are given directly by the table.

As a further example consider the IQL plan for $p_o = 0.05$, $p_1 = 0.03$, $\gamma = 1$. and $N = 1000$, which give $(n,c)=(113,5)$. The table directly gives $P(0.015) = 0.993$, $P(0.02) = 0.974$, $P(0.03) = 0.875$, and $P(0.05) = 0.50$. The formula gives $p_{95} = 2.613/111.8 = 0.0234$ and $p_{10} = 9.275/115.1 = 0.0806$. Furthermore the relation $P(p_1) + P(p_2) \cong 1$ may be used to obtain the three approximate values of $P(p_2)$ corresponding to the given values of $P(p_1)$ since p_2 has been tabulated as function of p_o and p_1 on p. 31 in the Appendix.

It should finally be noted that in case c and p are known, the formula may be used to find n as

$$n = \frac{m}{p} - \frac{m - c}{2} .$$

10. A generalization of the AOQL system of sampling inspection plans.

Suppose that the quality distribution of the main part of lots submitted for inspection is a binomial distribution with parameter p_1 and that the prior distribution otherwise is unknown. Suppose further that the cost functions are $k_s(p) = S_1$, $k_r(p) = R_1$, and $k_a(p) = A_2p$. The average costs for lots of quality p due to accepted defective items then become $(1-n/N)A_2pP(p)$ per item of the lot. In an attempt to control the damage resulting from accepted defective items one might specify an upper limit, k_L say, for these costs instead of choosing p_2 and $P(p_2)$ as in the first part of section 6.

As a reasonable principle for determining a sampling plan one may then choose to minimize the average costs for lots of normal quality, i.e. $R = n + (N-n)\gamma Q(p_1)$ where $\gamma = (R_1 - A_2p_1)/(S_1 - A_2p_1)$, under the restriction that $\max_p \{(1 - n/N)A_2pP(p)\} = k_L$.

One of the advantages of this system as compared to the LTPD system is that the (arbitrary) choice of two parameters, viz. p_2 and $P(p_2)$, is replaced by the choice of one parameter, k_L, which in most cases also will be more meaningful. Furthermore the system has the property that both the producer's and the consumer's risks tend to zero for $n \rightarrow \infty$.

It will be seen that $k_L/A_2 = p_L$ is identical to the AOQL in Dodge and Romig's terminology [3]. For $\gamma = 1$ we obtain the Dodge-Romig AOQL system.

The asymptotic properties of the present system are therefore identical to the properties of the AOQL system, see Hald and Kousgaard [9], with one addition which takes into account that γ may be different from 1. Comparing the proof in [9] with the corresponding one for the LTPD system given in section 5 it follows immediately that the result regarding γ is valid in both cases, i.e. the sampling plan for lot size N and cost parameter γ is for large N equal to the plan for lot size $N^* = N\gamma$ and cost parameter 1. It is conjectured that this property holds also for small N with good approximation if only $\gamma < 3$. The Dodge-Romig AOQL tables may therefore be used in such cases.

If rejected lots are rectified p_L has the usual AOQL interpretation. In cases with unknown A_2 and rectification one may therefore specify p_L and minimize R with $\gamma = R_1/S_1$ since A_2p_1 normally is small.

11. General remarks.

We shall here compare the three systems of sampling plans and the Bayesian solution under the assumption that $p_1 < p_r < p_2$. Furthermore some comments on the three systems are given for the case where p_r is unknown because one of the components, $k_a(p)$ or $k_r(p)$, of the cost function is unknown. We shall, however, always assume that p_1 represents a satisfactory and p_2 an unsatisfactory quality level so that lots of these qualities ideally should be accepted and rejected, respectively.

For a given prior distribution and given costs the optimum solution is the Bayesian one which for small N often will be acceptance (or rejection) without inspection. However, if the assumption of a stable prior distribution fails and there is no inspection heavy losses may be incurred before the change will be detected. Therefore a need exists for supplementing the Bayesian solution for small N with a sampling plan or for replacing the Bayesian solution in general by a system with similar properties as the Bayesian for large N and leading to a reasonable sampling plan also for small N. It follows from the discussion in section 8 that the IQL system has the desired properties, i.e. <u>the IQL plan with parameters</u> $(p_o, p_1, \gamma_1 + \gamma_2)$ <u>is recommended as a substitute for the Bayesian solution with parameters</u> $(p_1, p_2, \gamma_1, \gamma_2)$ <u>if the possibility of a deterioration of the prior distribution has to be taken into account.</u>

The LTPD and the AQL system may also be used for small lots but not for large lots since the economic efficiency of these plans as compared to the IQL and Bayesian plans tends to zero for $N \rightarrow \infty$. This is due to the fact that the fixed risk introduces a term proportional to N, $0.1\gamma_2 N$ and $0.05\gamma_1 N$ respectively, into the average costs and this term will for large N dominate over the sampling inspection costs $n = O(\ln N)$ and the remaining decision losses which tend to a constant, see section 5. <u>From an economic point of view it is therefore not advisable to use the</u>

LTPD and the AQL system for large lots. These systems will tend to give too small sample sizes and too large costs because of the fixed risk.

In view of the conclusion above it seems reasonable to try to reformulate the ideas behind the LTPD and the AQL system so that the fixed risk required only becomes of importance for small lots. This might be done by minimizing the average costs under the restriction $P(p_2) = \beta$ for $N \leq N_o$ and $P(p_2) = \beta N_o/N$ for $N > N_o$ (or similarly $Q(p_1) = \alpha$ for $N \leq N_o$ and $Q(p_1) = \alpha N_o/N$ for $N > N_o$). Theory and tables for such plans may easily be developed along similar lines as in the present paper, but they have the drawback of depending on two arbitrary parameters, (β, N_o) or (α, N_o).

It is possibly not worth while pursuing this idea further because a similar effect may be obtained by switching over from the LTPD (or AQL) system to the IQL system for a certain value of N, N_o say. If for some specific reason an upper limit of 10% has been fixed for the consumer's risk one may use the LTPD system for $N \leq N_o$ and the IQL system for $N > N_o$ where N_o is determined so that IQL plans for $N > N_o$ all have $P(p_2) < 0.10$.

As discussed previously another reason for introducing restrictions on the Bayes solution may be lack of detailed knowledge of one of the cost components, $k_a(p)$ or $k_r(p)$. This does not, however, change the results of the above discussion if only it is clear that p_1 represents a satisfactory and p_2 an unsatisfactory quality level. Since the economic consequences of wrong decisions are more serious for large than for small lots it is necessary that the risk of wrong decisions decreases with increasing lot size. This condition is satisfied by the IQL system but not by the LTPD and AQL systems.

Acknowledgements.

My thanks are due to Mr. Egon Jensen for making the program for the computer, to Mr. E. Kousgaard for checking the formulas, investigating approximations, and carrying out a lot of exploratory computations, and to Mr. P. Thyregod for assistance in the final revision of the manuscript.

References.

1. D. Blackwell and J.L. Hodges: The probability in the extreme tail of a convolution. Ann. Math. Stat. 30, 1959, 1113-1120.

2. H.F. Dodge and H.G. Romig: A method of sampling inspection. Bell System Technical Journal, 7, 1929, 613-631. Reprinted in H.F. Dodge and H.G. Romig: Sampling inspection tables, John Wiley, New York, 2. ed., 1959.

3. H.F. Dodge and H.G. Romig: Single sampling and double sampling inspection tables. Bell System Technical Journal, 20, 1941, 1-61. Reprinted in the book mentioned under 2.

4. H.F. Dodge: A general procedure for sampling inspection by attributes - based on the AQL concept. ASQC Annual Convention Transactions, 1963, 7-19.

5. R.A. Fisher and E.A. Cornish: The percentile points of distributions having known cumulants. Technometrics, 2, 1960, 209-225.

6. A. Hald: Some limit theorems for the Dodge-Romig LTPD single sampling inspection plans. Technometrics, 4, 1962, 497-513.

7. A. Hald: Single sampling inspection plans with specified acceptance probability and minimum costs. Duplicated report, Copenhagen, 1963.

8. A. Hald: Bayesian single sampling attribute plans for discrete prior distributions. Duplicated report, Copenhagen, 1964.

9. A. Hald and E. Kousgaard: Some limit theorems for the Dodge-Romig AOQL single sampling inspection plans. Sankhya, A, 25, 1963, 255-268.

10. Kungl. Arméfygförvaltningen: Provtagningstabeller för statistisk felantalskontroll. Stockholm, 1959.

11. N. Markbäck: Bell-Kontroll. Tabeller för enkel Provtagning. Tekniskt Meddelande, Nr. 7a V. Sveriges Mekanförbund, Stockholm, 1950.

12. Statistical Research Group, Columbia University: Sampling inspection. McGraw-Hill, New York, 1948.

13. I. Weibull: A method of determining inspection plans on an economic basis. Bull. Intern. Stat. Inst., 33, 1951, 85-104.

Appendix of Tables

LTPD single sampling tables with consumer's risk of 10 % and minimum average costs.

The tables on pp. 4 - 13 are based on a binomial consumer's risk of 10 %, $P(p_2) = 0.10$, and a binomial producer's risk, $Q(p_1) = 1 - P(p_1)$. The sampling plans given minimize the average costs $R_o = n + (N-n)\gamma Q(p_1)$.

The same plans minimize the average costs $R = n + (N - n)(\gamma_1 Q(p_1) + \gamma_2 P(p_2))$ for $P(p_2) = 0.10$ since $R = (1 - 0.1\gamma_2)\, R_o + 0.1\gamma_2 N$ with $\gamma = \gamma_1/(1 - 0.1\gamma_2)$.

The condition $P(p_2) = 0.10$ has been fulfilled as nearly as possible in the way that n has been determined as the smallest integer satisfying $B(c,n,p_2) \leq 0.10$.

The tables give n,c and 100 $P(p_1)$ as functions of N for γ = 1 and 5, and for the following 50 combinations of 100 p_2 and 100 p_1:

$100p_2$	$100p_1$				
0.5	0.05	0.1	0.15	0.2	0.25
1	0.1	0.2	0.3	0.4	0.5
2	0.2	0.4	0.6	0.8	1.0
3	0.3	0.6	0.9	1.2	1.5
4	0.8	1.2	1.6	2.0	2.4
5	1.0	1.5	2.0	2.5	3.0
7	2.1	2.8	3.5	4.2	4.9
10	3.0	4.0	5.0	6.0	7.0
15	4.5	6.0	7.5	9.0	10.5
20	6.0	8.0	10.0	12.0	14.0

Methods of interpolation have been discussed in section 6.

The tables may be used for $\gamma \neq 1$ and $\gamma \neq 5$ in the following way: For $\gamma \leq 3$ compute $N^* = N\gamma$ and use the plan for N^* and $\gamma = 1$. For $3 < \gamma < 10$ compute $N^* = N\gamma/5$ and use the plan for N^* and $\gamma = 5$.

For $\gamma > 1$ total inspection is cheaper than sampling inspection if $Q(p_1) > 1/\gamma$. In such cases the letter t has been added after the sample size.

If the consumer's risk is defined as a hypergeometric instead of a binomial probability a good approximation to the solution may be obtained by using the binomial c and correcting the binomial n to $n_h = n[1 - (np_2 - c)/(2Np_2)]$.

The tables on pp. 14 - 16 are based on the same assumptions with the only modification that the consumer's and the producer's risks have been computed from Poisson probabilities. The functions $m = np_2$ and $M = Np_2$ have been tabulated for $M < 50{,}000$ with c and $r = p_1/p_2$ as arguments for $c \leq 99$ and $r = 0.05, 0.10, \ldots, 0.70$, and for $\gamma = 1$ and 5. The optimum plan is (c,m) for $M(c-1) < M < M(c)$.

For $\gamma \leq 3$ use $M^* = M\gamma$ and the table for $\gamma = 1$. For $3 < \gamma < 10$ use $M^* = M\gamma/5$ and the table for $\gamma = 5$.

Underlining of M in the table for $\gamma = 5$ means that total inspection is cheaper than sampling inspection.

An approximation to the "binomial solution" may be obtained by using c from the Poisson table and correcting the corresponding n to $n_b = n - (np_2 - c)/2$.

Single Sampling Tables for LTPD = 0.5 per cent and γ =1

$100p_1$	0.05			0.10			0.15			0.20			0.25		
N	n	c	100P	n	c	100P	n	c	100P	n	c	100P	n	c	100P
300	All	-	-	All	-	-	All	-	-	All	-	-	All	-	-
500	460	0	79.4	460	0	63.1	460	0	50.1	460	0	39.8	460	0	31.6
700	460	0	79.4	460	0	63.1	460	0	50.1	460	0	39.8	460	0	31.6
1000	460	0	79.4	460	0	63.1	460	0	50.1	460	0	39.8	460	0	31.6
2000	460	0	79.4	777	1	81.7	777	1	67.5	777	1	54.0	777	1	42.1
3000	777	1	94.2	777	1	81.7	1063	2	78.5	1063	2	64.3	1063	2	50.4
5000	777	1	94.2	1063	2	90.8	1335	3	85.7	1597	4	78.2	1597	4	63.0
7000	777	1	94.2	1335	3	95.3	1597	4	90.5	1853	5	82.9	2105	6	72.3
10000	1063	2	98.3	1335	3	95.3	1853	5	93.7	2352	7	89.6	2839	9	82.1
20000	1063	2	98.3	1597	4	97.7	2352	7	97.2	3079	10	95.1	4023	14	91.4
30000	1335	3	99.5	1853	5	98.8	2597	8	98.2	3317	11	96.2	4718	17	94.5
50000	1335	3	99.5	2105	6	99.4	2839	9	98.8	4063	14	98.2	5406	20	96.5
70000	1335	3	99.5	2105	6	99.4	3079	10	99.2	4296	15	98.6	6087	23	97.9
100000	1597	4	99.9	2352	7	99.7	3317	11	99.5	4718	17	99.2	6539	25	98.3
200000	1597	4	99.9	2597	8	99.9	3554	12	99.7	5178	19	99.5	7659	30	99.2

Single Sampling Tables for LTPD = 1.0 per cent and γ =1

$100p_1$	0.10			0.20			0.30			0.40			0.50		
N	n	c	100P	n	c	100P	n	c	100P	n	c	100P	n	c	100P
200	All	-	-	All	-	-	All	-	-	All	-	-	All	-	-
300	230	0	79.4	230	0	63.1	230	0	50.1	230	0	39.8	230	0	31.6
500	230	0	79.4	230	0	63.1	230	0	50.1	230	0	39.8	230	0	31.6
700	230	0	79.4	230	0	63.1	230	0	50.1	230	0	39.8	230	0	31.6
1000	230	0	79.4	388	1	81.7	388	1	67.6	388	1	54.0	388	1	42.2
2000	388	1	94.2	531	2	90.8	531	2	78.5	667	3	72.1	667	3	57.2
3000	388	1	94.2	531	2	90.8	667	3	85.7	798	4	78.2	926	5	68.1
5000	531	2	98.3	667	3	95.4	926	5	93.7	1175	7	89.6	1418	9	82.2
7000	531	2	98.3	798	4	97.7	1051	6	95.8	1297	8	91.9	1658	11	86.7
10000	531	2	98.3	798	4	97.7	1175	7	97.2	1538	10	95.1	2010	14	91.4
20000	667	3	99.5	926	5	98.8	1297	8	98.2	1893	13	97.7	2587	19	95.9
30000	667	3	99.5	1051	6	99.4	1418	9	98.8	2010	14	98.2	2929	22	97.4
50000	798	4	99.9	1175	7	99.7	1658	11	99.5	2242	16	98.9	3268	25	98.4
70000	798	4	99.9	1175	7	99.7	1776	12	99.7	2473	18	99.4	3604	28	99.0
100000	798	4	99.9	1297	8	99.9	1776	12	99.7	2587	19	99.5	3808	30	99.2
200000	926	5	100.0	1418	9	99.9	2010	14	99.9	2929	22	99.8	4383	35	99.7

Single Sampling Tables for LTPD = 2.0 per cent and γ = 1

$100p_1$	0.20			0.40			0.60			0.80			1.00		
N	n	c	100P	n	c	100P	n	c	100P	n	c	100P	n	c	100P
100	All	-	-	All	-	-	All	-	-	All	-	-	All	-	-
200	114	0	79.6	114	0	63.3	114	0	50.4	114	0	40.0	114	0	31.8
300	114	0	79.6	114	0	63.3	114	0	50.4	114	0	40.0	114	0	31.8
500	114	0	79.6	194	1	81.8	194	1	67.5	194	1	54.0	194	1	42.1
700	194	1	94.2	194	1	81.8	265	2	78.6	265	2	64.4	265	2	50.5
1000	194	1	94.2	265	2	90.9	265	2	78.6	333	3	72.2	333	3	57.3
2000	265	2	98.3	333	3	95.4	398	4	90.6	525	6	86.8	587	7	76.2
3000	265	2	98.3	333	3	95.4	462	5	93.8	587	7	89.7	768	10	84.7
5000	265	2	98.3	398	4	97.7	587	7	97.3	768	10	95.2	945	13	90.2
7000	333	3	99.5	462	5	98.8	587	7	97.3	828	11	96.2	1120	16	93.7
10000	333	3	99.5	462	5	98.8	648	8	98.2	945	13	97.8	1292	19	96.0
20000	398	4	99.9	525	6	99.4	768	10	99.2	1120	16	99.0	1519	23	97.8
30000	398	4	99.9	587	7	99.7	828	11	99.5	1177	17	99.2	1688	26	98.6
50000	398	4	99.9	648	8	99.9	887	12	99.7	1292	19	99.5	1912	30	99.3
70000	462	5	100.0	648	8	99.9	945	13	99.8	1406	21	99.7	2023	32	99.5
100000	462	5	100.0	708	9	99.9	1004	14	99.9	1463	22	99.8	2134	34	99.6
200000	462	5	100.0	768	10	100.0	1120	16	99.9	1632	25	99.9	2410	39	99.8

Single Sampling Tables for LTPD = 3.0 per cent and γ = 1

$100p_1$	0.30			0.60			0.90			1.20			1.50		
N	n	c	100P	n	c	100P	n	c	100P	n	c	100P	n	c	100P
70	All	-	-	All	-	-	All	-	-	All	-	-	All	-	-
100	76	0	79.6	76	0	63.3	76	0	50.3	76	0	40.0	76	0	31.7
200	76	0	79.6	76	0	63.3	76	0	50.3	76	0	40.0	76	0	31.7
300	76	0	79.6	76	0	63.3	129	1	67.7	129	1	54.1	129	1	42.2
500	129	1	94.2	129	1	81.8	176	2	78.8	176	2	64.6	176	2	50.7
700	129	1	94.2	176	2	91.0	176	2	78.8	221	3	72.5	221	3	57.1
1000	129	1	94.2	176	2	91.0	221	3	86.0	265	4	78.5	349	6	72.8
2000	176	2	98.4	221	3	95.5	308	5	93.8	390	7	89.9	511	10	84.9
3000	176	2	98.4	265	4	97.7	349	6	96.0	471	9	93.9	629	13	90.4
5000	221	3	99.5	308	5	98.9	431	8	98.3	551	11	96.3	784	17	94.7
7000	221	3	99.5	308	5	98.9	471	9	98.9	629	13	97.8	860	19	96.1
10000	221	3	99.5	349	6	99.4	471	9	98.9	668	14	98.3	936	21	97.1
20000	265	4	99.9	390	7	99.7	551	11	99.5	784	17	99.2	1124	26	98.7
30000	265	4	99.9	431	8	99.9	590	12	99.7	860	19	99.5	1236	29	99.2
50000	308	5	100.0	431	8	99.9	629	13	99.8	936	21	99.7	1385	33	99.6
70000	308	5	100.0	471	9	99.9	668	14	99.9	974	22	99.8	1458	35	99.7
100000	308	5	100.0	471	9	99.9	707	15	99.9	1049	24	99.9	1532	37	99.8
200000	349	6	100.0	511	10	100.0	784	17	100.0	1124	26	99.9	1715	42	99.9

Single Sampling Tables for LTPD = 4.0 per cent and $\gamma = 1$

$100p_1$	0.80			1.20			1.60			2.00			2.40		
N	n	c	100P	n	c	100P	n	c	100P	n	c	100P	n	c	100P
50	All	-	-	All	-	-	All	-	-	All	-	-	All	-	-
70	57	0	63.3	57	0	50.3	57	0	39.9	57	0	31.6	57	0	25.0
100	57	0	63.3	57	0	50.3	57	0	39.9	57	0	31.6	57	0	25.0
200	57	0	63.3	57	0	50.3	57	0	39.9	57	0	31.6	57	0	25.0
300	96	1	82.1	96	1	68.0	96	1	54.4	96	1	42.5	96	1	32.6
500	132	2	91.0	132	2	78.8	166	3	72.4	198	4	63.7	198	4	48.3
700	132	2	91.0	166	3	86.0	198	4	78.7	230	5	60.6	230	5	52.4
1000	166	3	95.5	198	4	90.8	230	5	83.4	292	7	76.7	353	9	65.7
2000	198	4	97.8	262	6	96.0	353	9	94.0	442	12	88.9	587	17	82.3
3000	198	4	97.8	292	7	97.4	383	10	95.3	530	15	92.9	701	21	87.4
5000	230	5	98.9	323	8	98.3	471	13	97.9	616	18	95.5	870	27	92.4
7000	262	6	99.4	353	9	98.9	501	14	98.3	701	21	97.2	1037	33	95.5
10000	262	6	99.4	383	10	99.3	530	15	98.7	786	24	98.2	1120	36	96.5
20000	292	7	99.7	442	12	99.7	616	18	99.4	898	28	99.1	1367	45	98.4
30000	323	8	99.9	471	13	99.8	673	20	99.7	982	31	99.4	1504	50	99.0
50000	353	9	99.9	501	14	99.9	701	21	99.7	1065	34	99.6	1612	54	99.3
70000	353	9	99.9	530	15	99.9	758	23	99.8	1120	36	99.7	1775	60	99.6
100000	383	10	100.0	559	16	99.9	786	24	99.9	1203	39	99.8	1883	64	99.7
200000	413	11	100.0	587	17	100.0	870	27	99.9	1313	43	99.9	2071	71	99.8

Single Sampling Tables for LTPD = 5.0 per cent and $\gamma = 1$

$100p_1$	1.00			1.50			2.00			2.50			3.00		
N	n	c	100P	n	c	100P	n	c	100P	n	c	100P	n	c	100P
30	All	-	-	All	-	-	All	-	-	All	-	-	All	-	-
50	45	0	63.6	45	0	50.7	45	0	40.3	45	0	32.0	45	0	25.4
70	45	0	63.6	45	0	50.7	45	0	40.3	45	0	32.0	45	0	25.4
100	45	0	63.6	45	0	50.7	45	0	40.3	45	0	32.0	45	0	25.4
200	77	1	82.0	77	1	67.9	77	1	54.3	77	1	42.3	77	1	32.4
300	77	1	82.0	105	2	79.1	105	2	64.9	105	2	51.0	105	2	38.7
500	105	2	91.1	132	3	86.2	158	4	78.9	158	4	63.9	158	4	48.5
700	132	3	95.6	158	4	90.9	184	5	83.5	209	6	73.0	258	8	62.9
1000	132	3	95.6	184	5	94.0	209	6	87.2	282	9	82.8	306	10	68.6
2000	158	4	97.8	209	6	96.0	306	10	95.4	400	14	91.9	492	18	83.9
3000	184	5	98.9	258	8	98.3	353	12	97.3	446	16	94.1	628	24	90.4
5000	209	6	99.5	282	9	98.9	400	14	98.4	538	20	96.8	740	29	93.7
7000	209	6	99.5	306	10	99.3	423	15	98.8	583	22	97.7	873	35	96.3
10000	234	7	99.7	330	11	99.5	446	16	99.0	628	24	98.3	962	39	97.3
20000	258	8	99.9	353	12	99.7	515	19	99.6	740	29	99.2	1137	47	98.7
30000	258	8	99.9	377	13	99.8	538	20	99.7	807	32	99.5	1267	53	99.2
50000	282	9	99.9	400	14	99.9	583	22	99.8	873	35	99.7	1397	59	99.6
70000	282	9	99.9	423	15	99.9	628	24	99.9	940	38	99.8	1440	61	99.6
100000	306	10	100.0	446	16	99.9	651	25	99.9	984	40	99.9	1548	66	99.8
200000	330	11	100.0	492	18	100.0	718	28	100.0	1071	44	99.9	1720	74	99.9

Single Sampling Tables for LTPD = 7.0 per cent and $\gamma = 1$

$100p_1$	2.10			2.80			3.50			4.20			4.90		
N	n	c	100P	n	c	100P	n	c	100P	n	c	100P	n	c	100P
30	All	-	-	All	-	-	All	-	-	All	-	-	All	-	-
50	32	0	50.7	32	0	40.3	32	0	32.0	32	0	25.3	32	0	20.0
70	32	0	50.7	32	0	40.3	32	0	32.0	32	0	25.3	32	0	20.0
100	32	0	50.7	32	0	40.3	32	0	32.0	32	0	25.3	32	0	20.0
200	75	2	79.1	75	2	64.9	75	2	50.9	75	2	38.5	75	2	28.2
300	75	2	79.1	94	3	73.0	94	3	58.1	94	3	44.0	94	3	31.8
500	113	4	91.0	131	5	83.7	166	7	77.3	166	7	60.3	166	7	43.0
700	131	5	94.1	166	7	90.4	201	9	83.1	218	10	69.0	268	13	55.8
1000	149	6	96.1	184	8	92.4	235	11	87.5	301	15	79.9	334	17	62.6
2000	166	7	97.5	235	11	96.6	301	15	93.4	399	21	87.9	575	32	80.1
3000	184	8	98.4	268	13	98.0	367	19	96.4	512	28	93.4	701	40	85.8
5000	218	10	99.3	301	15	98.8	399	21	97.4	591	33	95.8	919	54	92.3
7000	218	10	99.3	318	16	99.1	464	25	98.6	654	37	97.0	1027	61	94.3
10000	235	11	99.6	334	17	99.3	496	27	99.0	748	43	98.3	1150	69	96.0
20000	268	13	99.8	383	20	99.7	575	32	99.6	857	50	99.1	1425	87	98.2
30000	285	14	99.9	399	21	99.8	607	34	99.7	950	56	99.5	1577	97	98.9
50000	301	15	99.9	448	24	99.9	654	37	99.8	1027	61	99.7			
70000	318	16	100.0	464	25	99.9	701	40	99.9	1104	66	99.8			
100000	334	17	100.0	480	26	99.9	717	41	99.9	1150	69	99.8			
200000	351	18	100.0	528	29	100.0	779	45	99.9	1242	75	99.9			

Single Sampling Tables for LTPD = 10.0 per cent and $\gamma = 1$

$100p_1$	3.00			4.00			5.00			6.00			7.00		
N	n	c	100P	n	c	100P	n	c	100P	n	c	100P	n	c	100P
30	22	0	51.2	22	0	40.7	22	0	32.4	22	0	25.6	22	0	20.3
50	22	0	51.2	22	0	40.7	22	0	32.4	22	0	25.6	22	0	20.3
70	22	0	51.2	22	0	40.7	22	0	32.4	22	0	25.6	22	0	20.3
100	38	1	68.4	38	1	54.8	38	1	42.7	38	1	32.6	22	0	20.3
200	52	2	79.5	65	3	73.8	65	3	59.0	65	3	44.8	65	3	32.5
300	65	3	86.9	78	4	79.8	91	5	69.6	91	5	53.3	91	5	38.1
500	91	5	94.4	116	7	90.6	140	9	83.6	175	12	74.7	175	12	54.6
700	91	5	94.4	128	8	92.8	175	12	89.9	210	15	80.4	233	17	63.3
1000	116	7	97.6	140	9	94.5	187	13	91.3	233	17	83.5	312	24	72.9
2000	128	8	98.5	175	12	97.6	233	17	95.4	346	27	93.2	511	42	87.7
3000	140	9	99.0	199	14	98.5	267	20	97.2	390	31	95.3	609	51	91.8
5000	152	10	99.4	210	15	98.9	312	24	98.5	468	38	97.5	728	62	95.0
7000	164	11	99.6	233	17	99.4	346	27	99.1	511	42	98.3	814	70	96.5
10000	175	12	99.7	256	19	99.6	368	29	99.3	544	45	98.7	889	77	97.5
20000	187	13	99.8	267	20	99.7	413	33	99.7	631	53	99.4	1070	94	98.9
30000	210	15	99.9	301	23	99.9	446	36	99.8	674	57	99.6			
50000	210	15	99.9	312	24	99.9	479	39	99.9	728	62	99.7			
70000	233	17	100.0	335	26	99.9	511	42	99.9	771	66	99.8			
100000	233	17	100.0	346	27	100.0	522	43	99.9	814	70	99.9			
200000	256	19	100.0	379	30	100.0	566	47	100.0	889	77	99.9			

Single Sampling Tables for LTPD = 15.0 per cent and γ =1

$100p_1$	4.50			6.00			7.50			9.00			10.50		
N	n	c	100P	n	c	100P	n	c	100P	n	c	100P	n	c	100P
30	15	0	50.1	15	0	39.5	15	0	31.1	15	0	24.3	15	0	18.9
50	15	0	50.1	15	0	39.5	15	0	31.1	15	0	24.3	15	0	18.9
70	25	1	68.9	25	1	55.3	25	1	43.1	25	1	32.9	25	1	24.6
100	34	2	80.4	34	2	66.6	34	2	52.6	34	2	39.9	34	2	29.3
200	43	3	87.3	60	5	85.0	68	6	75.3	68	6	58.6	68	6	42.0
300	60	5	94.8	68	6	88.7	85	8	81.4	100	10	71.2	100	10	51.7
500	68	6	96.7	85	8	93.1	108	11	88.9	139	15	81.5	154	17	64.8
700	68	6	96.7	100	10	96.2	124	13	91.8	154	17	84.7	229	27	77.6
1000	77	7	97.8	108	11	97.1	154	17	95.9	192	22	90.3	266	32	82.1
2000	100	10	99.5	124	13	98.3	177	20	97.5	266	32	96.2	368	46	90.7
3000	100	10	99.5	139	15	99.1	192	22	98.2	288	35	97.2	433	55	93.9
5000	108	11	99.7	154	17	99.5	229	27	99.2	339	42	98.6	519	67	96.6
7000	116	12	99.8	162	18	99.6	229	27	99.2	368	46	99.0	569	74	97.5
10000	116	12	99.8	177	20	99.8	244	29	99.5	397	50	99.3	640	84	98.5
20000	139	15	99.9	192	22	99.9	288	35	99.8	433	55	99.6	725	96	99.2
30000	139	15	99.9	207	24	99.9	310	38	99.9	469	60	99.8			
50000	154	17	100.0	222	26	100.0	332	41	99.9	512	65	99.9			
70000	154	17	100.0	229	27	100.0	339	42	99.9	519	67	99.9			
100000	154	17	100.0	229	27	100.0	361	45	100.0	569	74	99.9			
200000	177	20	100.0	244	29	100.0	383	48	100.0	612	80	100.0			

Single Sampling Tables for LTPD = 20.0 per cent and γ=1

$100p_1$	6.00			8.00			10.00			12.00			14.00		
N	n	c	100P	n	c	100P	n	c	100P	n	c	100P	n	c	100P
30	11	0	50.6	11	0	40.0	11	0	31.4	11	0	24.5	11	0	19.0
50	18	1	70.6	18	1	57.2	18	1	45.0	18	1	34.6	18	1	26.0
70	18	1	70.6	25	2	67.7	25	2	53.7	25	2	40.9	18	1	26.0
100	25	2	81.3	32	3	74.9	38	4	67.0	38	4	51.4	38	4	37.0
200	38	4	92.5	51	6	89.0	57	7	79.3	63	8	65.8	86	12	57.1
300	38	4	92.5	57	7	91.7	69	9	85.2	86	12	77.2	109	16	64.5
500	51	6	96.8	69	9	95.3	86	12	91.4	126	19	88.3	154	24	75.7
700	57	7	98.0	75	10	96.4	109	16	95.7	143	22	91.2	193	31	82.5
1000	63	8	98.7	86	12	98.1	109	16	95.7	154	24	92.8	226	37	86.8
2000	69	9	99.2	98	14	98.9	143	22	98.5	204	33	97.0	318	54	94.4
3000	75	10	99.5	109	16	99.4	154	24	98.9	226	37	98.0	361	62	96.2
5000	86	12	99.8	115	17	99.5	171	27	99.4	253	42	98.8	409	71	97.6
7000	86	12	99.8	126	19	99.8	182	29	99.6	275	46	99.2	457	80	98.5
10000	92	13	99.9	126	19	99.8	193	31	99.7	291	49	99.4	489	86	98.9
20000	98	14	99.9	143	22	99.9	215	35	99.8	334	57	99.7			
30000	109	16	100.0	154	24	99.9	226	37	99.9	361	62	99.8			
50000	109	16	100.0	160	25	100.0	253	42	100.0	377	65	99.9			
70000	115	17	100.0	171	27	100.0	253	42	100.0	393	68	99.9			
100000	121	18	100.0	171	27	100.0	264	44	100.0	425	74	100.0			
200000	126	19	100.0	193	31	100.0	291	49	100.0	457	80	100.0			

Single Sampling Tables for LTPD = 0.5 per cent and $\gamma = 5$

$100p_1$	0.05			0.10			0.15			0.20			0.25		
N	n	c	100P	n	c	100P	n	c	100P	n	c	100P	n	c	100P
300	All	-	-	All	-	-	All	-	-	All	-	-	All	-	-
500	460t	0	79.4	460t	0	63.1	460t	0	50.1	460t	0	39.8	460t	0	31.6
700	460t	0	79.4	460t	0	63.1	460t	0	50.1	460t	0	39.8	460t	0	31.6
1000	777	1	94.2	777	1	81.7	777t	1	67.5	777t	1	54.0	777t	1	42.1
2000	777	1	94.2	1335	3	95.3	1597	4	90.5	1853	5	82.9	1853t	5	68.0
3000	1063	2	98.3	1335	3	95.3	1853	5	93.7	2352	7	89.6	2839	9	82.1
5000	1063	2	98.3	1597	4	97.7	2105	6	95.8	2839	9	93.7	3554	12	88.4
7000	1335	3	99.5	1853	5	98.8	2352	7	97.2	3317	11	96.2	4256	15	92.6
10000	1335	3	99.5	1853	5	98.8	2839	9	98.8	3789	13	97.7	4948	18	95.2
20000	1597	4	99.9	2352	7	99.7	3079	10	99.2	4488	16	98.9	6313	24	98.1
30000	1597	4	99.9	2352	7	99.7	3554	12	99.7	4948	18	99.4	6988	27	98.8
50000	1597	4	99.9	2597	8	99.9	3789	13	99.8	5406	20	99.6	7882	31	99.3
70000	1853	5	100.0	2839	9	99.9	4023	14	99.9	5634	21	99.7	8327	33	99.5
100000	1853	5	100.0	2839	9	99.9	4256	15	99.9	6087	23	99.8	8992	36	99.7
200000	2105	6	100.0	3079	10	100.0	4488	16	99.9	6764	26	99.9	10094	41	99.9

Single Sampling Tables for LTPD = 1.0 per cent and $\gamma = 5$

$100p_1$	0.10			0.20			0.30			0.40			0.50		
N	n	c	100P	n	c	100P	n	c	100P	n	c	100P	n	c	100P
200	All	-	-	All	-	-	All	-	-	All	-	-	All	-	-
300	230t	0	79.4	230t	0	63.1	230t	0	50.1	230t	0	39.8	230t	0	31.6
500	388	1	94.2	388	1	81.7	388t	1	67.6	388t	1	54.0	388t	1	42.2
700	388	1	94.2	531	2	90.8	667	3	85.7	667t	3	72.1	667t	3	57.2
1000	388	1	94.2	667	3	95.4	798	4	90.5	926	5	83.0	926t	5	68.1
2000	531	2	98.3	798	4	97.7	1051	6	95.8	1297	8	91.9	1658	11	86.7
3000	667	3	99.5	926	5	98.8	1175	7	97.2	1538	10	95.1	2010	14	91.4
5000	667	3	99.5	926	5	98.8	1418	9	98.8	1893	13	97.7	2473	18	95.3
7000	667	3	99.5	1051	6	99.4	1418	9	98.8	2010	14	98.2	2815	21	97.0
10000	798	4	99.9	1175	7	99.7	1538	10	99.2	2242	16	98.9	3155	24	98.1
20000	798	4	99.9	1297	8	99.9	1776	12	99.7	2587	19	99.5	3716	29	99.1
30000	926	5	100.0	1297	8	99.9	1893	13	99.8	2815	21	99.7	4050	32	99.4
50000	926	5	100.0	1418	9	99.9	2127	15	99.9	3042	23	99.8	4494	36	99.7
70000	926	5	100.0	1538	10	100.0	2127	15	99.9	3155	24	99.9	4715	38	99.8
100000	1051	6	100.0	1538	10	100.0	2242	16	99.9	3380	26	99.9	5045	41	99.9
200000	1051	6	100.0	1658	11	100.0	2473	18	100.0	3604	28	100.0	5483	45	99.9

Single Sampling Tables for LTPD = 2.0 per cent and γ =5

$100p_1$	0.20			0.40			0.60			0.80			1.00		
N	n	c	100P	n	c	100P	n	c	100P	n	c	100P	n	c	100P
100	All	-	-	All	-	-	All	-	-	All	-	-	All	-	-
200	194	1	94.2	194	1	81.8	194t	1	67.5	194t	1	54.0	194t	1	42.1
300	194	1	94.2	265	2	90.9	265t	2	78.6	265t	2	64.4	265t	2	50.5
500	194	1	94.2	333	3	95.4	398	4	90.6	462	5	83.1	462t	5	68.3
700	265	2	98.3	333	3	95.4	462	5	93.8	525	6	86.8	648t	8	79.5
1000	265	2	98.3	398	4	97.7	525	6	95.9	648	8	92.0	828	11	86.8
2000	333	3	99.5	462	5	98.8	648	8	98.2	828	11	96.2	1120	16	93.7
3000	333	3	99.5	525	6	99.4	708	9	98.8	945	13	97.8	1349	20	96.6
5000	398	4	99.9	587	7	99.7	768	10	99.2	1120	16	99.0	1576	24	98.1
7000	398	4	99.9	587	7	99.7	828	11	99.5	1177	17	99.2	1688	26	98.6
10000	398	4	99.9	648	8	99.9	887	12	99.7	1292	19	99.5	1856	29	99.1
20000	462	5	100.0	708	9	99.9	1004	14	99.9	1463	22	99.8	2134	34	99.6
30000	462	5	100.0	708	9	99.9	1062	15	99.9	1519	23	99.8	2300	37	99.8
50000	525	6	100.0	768	10	100.0	1120	16	99.9	1632	25	99.9	2465	40	99.8
70000	525	6	100.0	828	11	100.0	1177	17	100.0	1744	27	99.9	2575	42	99.9
100000	525	6	100.0	828	11	100.0	1235	18	100.0	1800	28	100.0	2739	45	99.9
200000	587	7	100.0	887	12	100.0	1292	19	100.0	1968	31	100.0	2958	49	100.0

Single Sampling Tables for LTPD = 3.0 per cent and γ=5

$100p_1$	0.30			0.60			0.90			1.20			1.50		
N	n	c	100P	n	c	100P	n	c	100P	n	c	100P	n	c	100P
70	All	-	-	All	-	-	All	-	-	All	-	-	All	-	-
100	76t	0	79.6	76t	0	63.3	76t	0	50.3	76t	0	40.0	76t	0	31.7
200	129	1	94.2	176	2	91.0	176t	2	78.8	176t	2	64.6	176t	2	50.7
300	129	1	94.2	176	2	91.0	221	3	86.0	265t	4	78.5	265t	4	63.4
500	176	2	98.4	221	3	95.5	308	5	93.8	390	7	8[illegible].9	471	9	82.5
700	176	2	98.4	265	4	97.7	349	6	96.0	431	8	92.1	551	11	87.0
1000	221	3	99.5	308	5	98.9	390	7	97.3	511	10	95.3	668	14	91.7
2000	221	3	99.5	349	6	99.4	471	9	98.9	629	13	97.8	860	19	96.1
3000	265	4	99.9	349	6	99.4	511	10	99.3	707	15	98.7	1012	23	97.9
5000	265	4	99.9	390	7	99.7	551	11	99.5	822	18	99.4	1124	26	98.7
7000	265	4	99.9	431	8	99.9	590	12	99.7	860	19	99.5	1236	29	99.2
10000	308	5	100.0	431	8	99.9	629	13	99.8	898	20	99.6	1348	32	99.5
20000	308	5	100.0	471	9	99.9	707	15	99.9	1012	23	99.8	1532	37	99.8
30000	308	5	100.0	511	10	100.0	746	16	99.9	1087	25	99.9	1605	39	99.8
50000	349	6	100.0	551	11	100.0	784	17	100.0	1162	27	99.9	1715	42	99.9
70000	349	6	100.0	551	11	100.0	822	18	100.0	1199	28	100.0	1825	45	99.9
100000	349	6	100.0	590	12	100.0	860	19	100.0	1275	30	100.0	1898	47	100.0
200000	390	7	100.0	629	13	100.0	898	20	100.0	1348	32	100.0	2079	52	100.0

Single Sampling Tables for LTPD = 4.0 per cent and $\gamma = 5$

$100p_1$	0.80			1.20			1.60			2.00			2.40		
N	n	c	100P	n	c	100P	n	c	100P	n	c	100P	n	c	100P
50	All	-	-	All	-	-	All	-	-	All	-	-	All	-	-
70	57t	0	63.3	57t	0	50.3	57t	0	39.9	57t	0	31.6	57t	0	25.0
100	96	1	82.1	96t	1	68.0	96t	1	54.4	96t	1	42.5	96t	1	32.6
200	132	2	91.0	166	3	86.0	198t	4	78.7	198t	4	63.7	198t	4	48.3
300	166	3	95.5	198	4	90.8	262	6	87.0	292t	7	76.7	292t	7	59.8
500	198	4	97.8	262	6	96.0	323	8	92.2	413	11	87.1	471t	13	75.4
700	198	4	97.8	292	7	97.4	383	10	95.3	471	13	90.5	616	18	83.7
1000	230	5	98.9	323	8	98.3	413	11	96.4	559	16	93.9	758	23	89.3
2000	262	6	99.4	383	10	99.3	530	15	98.7	701	21	97.2	1037	33	95.5
3000	292	7	99.7	413	11	99.5	587	17	99.2	814	25	98.5	1203	39	97.3
5000	323	8	99.9	442	12	99.7	645	19	99.6	926	29	99.2	1367	45	98.4
7000	323	8	99.9	471	13	99.8	673	20	99.7	982	31	99.4	1504	50	99.0
10000	353	9	99.9	501	14	99.9	701	21	99.7	1037	33	99.6	1612	54	99.3
20000	383	10	100.0	559	16	99.9	786	24	99.9	1203	39	99.8	1856	63	99.7
30000	383	10	100.0	587	17	100.0	842	26	99.9	1258	41	99.9	1990	68	99.8
50000	413	11	100.0	616	18	100.0	898	28	100.0	1367	45	99.9	2178	75	99.9
70000	413	11	100.0	616	18	100.0	926	29	100.0	1422	47	100.0	2232	77	99.9
100000	442	12	100.0	645	19	100.0	954	30	100.0	1449	48	100.0	2339	81	99.9
200000	471	13	100.0	701	21	100.0	1037	33	100.0	1585	53	100.0	2526	88	100.0

Single Sampling Tables for LTPD = 5.0 per cent and $\gamma = 5$

$100p_1$	1.00			1.50			2.00			2.50			3.00		
N	n	c	100P	n	c	100P	n	c	100P	n	c	100P	n	c	100P
30	All	-	-	All	-	-	All	-	-	All	-	-	All	-	-
50	45t	0	63.6	45t	0	50.7	45t	0	40.3	45t	0	32.0	45t	0	25.4
70	45t	0	63.6	45t	0	50.7	45t	0	40.3	45t	0	32.0	45t	0	25.4
100	77	1	82.0	77t	1	67.9	77t	1	54.3	77t	1	42.3	77t	1	32.4
200	132	3	95.6	158	4	90.9	184	5	83.5	184t	5	68.7	184t	5	52.4
300	132	3	95.6	184	5	94.0	234	7	90.0	282	9	82.8	282t	9	65.9
500	158	4	97.8	209	6	96.0	282	9	94.1	353	12	89.1	469	17	82.5
700	184	5	98.9	234	7	97.4	330	11	96.4	423	15	93.1	561	21	87.4
1000	184	5	98.9	258	8	98.3	353	12	97.3	492	18	95.6	673	26	91.9
2000	209	6	99.5	306	10	99.3	446	16	99.0	628	24	98.3	873	35	96.3
3000	234	7	99.7	330	11	99.5	469	17	99.3	673	26	98.8	1027	42	98.0
5000	258	8	99.9	377	13	99.8	515	19	99.6	785	31	99.4	1180	49	98.9
7000	258	8	99.9	400	14	99.9	561	21	99.7	829	33	99.6	1267	53	99.2
10000	282	9	99.9	400	14	99.9	583	22	99.8	873	35	99.7	1332	56	99.4
20000	306	10	100.0	446	16	99.9	651	25	99.9	984	40	99.9	1548	66	99.8
30000	306	10	100.0	469	17	100.0	673	26	99.9	1027	42	99.9	1634	70	99.8
50000	330	11	100.0	492	18	100.0	740	29	100.0	1093	45	99.9	1784	77	99.9
70000	353	12	100.0	515	19	100.0	740	29	100.0	1137	47	100.0	1827	79	99.9
100000	353	12	100.0	538	20	100.0	785	31	100.0	1202	50	100.0	1891	82	99.9
200000	377	13	100.0	561	21	100.0	851	34	100.0	1289	54	100.0	2083	91	100.0

Single Sampling Tables for LTPD = 7.0 per cent and γ =5

$100p_1$	2.10			2.80			3.50			4.20			4.90		
N	n	c	100P	n	c	100P	n	c	100P	n	c	100P	n	c	100P
30	All	-	-	All	-	-	All	-	-	All	-	-	All	-	-
50	32t	0	50.7	32t	0	40.3	32t	0	32.0	32t	0	25.3	32t	0	20.0
70	55t	1	67.8	55t	1	54.2	55t	1	42.2	55t	1	32.2	55t	1	24.2
100	94	3	86.4	94t	3	73.0	94t	3	58.1	94t	3	44.0	94t	3	31.8
200	131	5	94.1	149	6	87.4	184	8	80.2	184t	8	63.1	184t	8	45.0
300	149	6	96.1	184	8	92.4	235	11	87.5	285t	14	77.9	285t	14	57.4
500	166	7	97.5	235	11	96.6	301	15	93.4	399	21	87.9	496t	27	75.2
700	184	8	98.4	268	13	98.0	334	17	95.1	464	25	91.4	638	36	83.2
1000	201	9	98.9	285	14	98.4	399	21	97.4	544	30	94.4	779	45	88.6
2000	235	11	99.6	334	17	99.3	480	26	98.8	701	40	97.7	1027	61	94.3
3000	252	12	99.7	367	19	99.6	528	29	99.3	779	45	98.6	1242	75	97.0
5000	268	13	99.8	399	21	99.8	575	32	99.6	888	52	99.2	1425	87	98.2
7000	285	14	99.9	399	21	99.8	607	34	99.7	950	56	99.5	1577	97	98.9
10000	301	15	99.9	448	24	99.9	638	36	99.8	1027	61	99.7			
20000	334	17	100.0	480	26	99.9	717	41	99.9	1150	69	99.8			
30000	334	17	100.0	512	28	100.0	748	43	99.9	1196	72	99.9			
50000	367	19	100.0	528	29	100.0	826	48	100.0	1303	79	99.9			
70000	383	20	100.0	560	31	100.0	857	50	100.0	1364	83	100.0			
100000	383	20	100.0	575	32	100.0	888	52	100.0	1410	86	100.0			
200000	416	22	100.0	607	34	100.0	950	56	100.0	1532	94	100.0			

Single Sampling Tables for LTPD = 10.0 per cent and γ=5

$100p_1$	3.00			4.00			5.00			6.00			7.00		
N	n	c	100P	n	c	100P	n	c	100P	n	c	100P	n	c	100P
30	22t	0	51.2	22t	0	40.7	22t	0	32.4	22t	0	25.6	22t	0	20.3
50	38t	1	68.4	38t	1	54.8	38t	1	42.7	38t	1	32.6	38t	1	24.5
70	65	3	86.9	65t	3	73.8	65t	3	59.0	65t	3	44.8	65t	3	32.5
100	78	4	91.5	91	5	84.3	91t	5	69.6	91t	5	53.3	91t	5	38.1
200	91	5	94.4	128	8	92.8	152	10	85.9	199t	14	78.3	199t	14	57.8
300	116	7	97.6	152	10	95.7	187	13	91.3	256	19	86.1	290t	22	70.2
500	128	8	98.5	175	12	97.6	233	17	95.4	312	24	91.2	446	36	83.7
700	140	9	99.0	187	13	98.1	267	20	97.2	379	30	94.8	522	43	88.2
1000	152	10	99.4	210	15	98.9	301	23	98.3	435	35	96.7	609	51	91.8
2000	175	12	99.7	256	19	99.6	357	28	99.2	522	43	98.4	846	73	97.0
3000	187	13	99.8	267	20	99.7	390	31	99.5	609	51	99.3	953	83	98.1
5000	199	14	99.9	290	22	99.8	424	34	99.7	674	57	99.6	1070	94	98.9
7000	210	15	99.9	301	23	99.9	446	36	99.8	685	58	99.6			
10000	210	15	99.9	312	24	99.9	468	38	99.9	728	62	99.7			
20000	233	17	100.0	346	27	100.0	511	42	99.9	814	70	99.9			
30000	245	18	100.0	368	29	100.0	544	45	100.0	857	74	99.9			
50000	256	19	100.0	379	30	100.0	577	48	100.0	921	80	100.0			
70000	267	20	100.0	390	31	100.0	609	51	100.0	953	83	100.0			
100000	267	20	100.0	413	33	100.0	620	52	100.0	1017	89	100.0			
200000	290	22	100.0	435	35	100.0	674	57	100.0	1070	94	100.0			

Single Sampling Tables for LTPD = 15.0 per cent and γ =5

$100p_1$	4.50			6.00			7.50			9.00			10.50		
N	n	c	100P	n	c	100P	n	c	100P	n	c	100P	n	c	100P
30	25t	1	68.9	25t	1	55.3	25t	1	43.1	25t	1	32.9	25t	1	24.6
50	43	3	87.3	43t	3	74.3	43t	3	59.5	43t	3	45.1	43t	3	32.6
70	52	4	91.6	60	5	85.0	68t	6	75.3	68t	6	58.6	68t	6	42.0
100	60	5	94.8	68	6	88.7	93	9	84.1	93t	9	67.4	93t	9	48.2
200	68	6	96.7	100	10	96.2	124	13	91.8	162	18	85.8	192t	22	71.6
300	85	8	98.6	108	11	97.1	154	17	95.9	192	22	90.3	266	32	82.1
500	100	10	99.5	124	13	98.3	177	20	97.5	244	29	94.9	361	45	90.2
700	100	10	99.5	139	15	99.1	192	22	98.2	288	35	97.2	397	50	92.3
1000	108	11	99.7	154	17	99.5	222	26	99.1	310	38	97.9	469	60	95.2
2000	116	12	99.8	177	20	99.8	244	29	99.5	368	46	99.0	619	81	98.2
3000	124	13	99.9	177	20	99.8	266	32	99.7	397	50	99.3	690	91	98.9
5000	139	15	99.9	192	22	99.9	288	35	99.8	462	59	99.7			
7000	139	15	99.9	207	24	99.9	310	38	99.9	469	60	99.8			
10000	154	17	100.0	222	26	100.0	325	40	99.9	512	66	99.9			
20000	154	17	100.0	229	27	100.0	354	44	100.0	562	73	99.9			
30000	162	18	100.0	244	29	100.0	368	46	100.0	569	74	99.9			
50000	177	20	100.0	259	31	100.0	397	50	100.0	626	82	100.0			
70000	177	20	100.0	266	32	100.0	397	50	100.0	640	84	100.0			
100000	192	22	100.0	281	34	100.0	419	53	100.0	676	89	100.0			
200000	192	22	100.0	288	35	100.0	433	55	100.0	725	96	100.0			

Single Sampling Tables for LTPD = 20.0 per cent and γ =5

$100p_1$	6.00			8.00			10.00			12.00			14.00		
N	n	c	100P	n	c	100P	n	c	100P	n	c	100P	n	c	100P
30	25	2	81.3	25t	2	67.7	25t	2	53.7	25t	2	40.9	25t	2	30.0
50	38	4	92.5	45	5	85.2	45t	5	70.8	45t	5	54.2	45t	5	38.4
70	38	4	92.5	51	6	89.0	63	8	82.5	69t	9	68.8	69t	9	49.5
100	51	6	96.8	65	8	93.7	75	10	87.4	98	14	80.6	98t	14	60.3
200	57	7	98.0	86	12	98.1	109	16	95.7	143	22	91.2	193	31	82.5
300	63	8	98.7	86	12	98.1	126	19	97.4	171	27	94.5	226	37	86.8
500	75	10	99.5	98	14	98.9	143	22	98.5	204	33	97.0	291	49	92.8
700	75	10	99.5	109	16	99.4	154	24	98.9	226	37	98.0	334	57	95.2
1000	86	12	99.8	115	17	99.5	171	27	99.4	253	42	98.8	393	68	97.2
2000	92	13	99.9	126	19	99.8	193	31	99.7	291	49	99.4	473	83	98.7
3000	98	14	99.9	143	22	99.9	204	33	99.8	318	54	99.6	526	93	99.2
5000	104	15	100.0	154	24	99.9	215	35	99.8	334	57	99.7			
7000	109	16	100.0	154	24	99.9	226	37	99.9	361	62	99.8			
10000	109	16	100.0	160	25	100.0	237	39	99.9	377	65	99.9			
20000	121	18	100.0	171	27	100.0	264	44	100.0	409	71	99.9			
30000	126	19	100.0	182	29	100.0	275	46	100.0	441	77	100.0			
50000	126	19	100.0	193	31	100.0	291	49	100.0	457	80	100.0			
70000	132	20	100.0	193	31	100.0	302	51	100.0	473	83	100.0			
100000	143	22	100.0	204	33	100.0	318	54	100.0	505	89	100.0			
200000	143	22	100.0	215	35	100.0	334	57	100.0	526	93	100.0			

Single Sampling Tables with Consumer's Risk of 10 %

$B(c,m) = 0.10$, $r = p_1/p_2$, $m = np_2$, $M = Np_2$, $\gamma = 1$.

c	r / m	0.70 M	0.65 M	0.60 M	0.55 M	0.50 M	0.45 M	0.40 M	0.35 M	0.30 M	0.25 M	0.20 M
0	2.303	10.9	10.0	9.43	8.99	8.67	8.47	8.36	8.36	8.48	8.75	9.28
1	3.890	14.9	13.9	13.3	12.9	12.7	12.7	12.9	13.3	14.1	15.6	18.2
2	5.322	18.5	17.5	16.9	16.7	16.7	17.1	17.8	19.1	21.4	25.5	33.6
3	6.681	21.9	21.0	20.5	20.5	20.9	21.8	23.5	26.4	31.4	41.0	62.0
4	7.994	25.3	24.5	24.2	24.5	25.4	27.2	30.3	35.6	45.4	65.7	116
5	9.275	28.7	28.0	28.0	28.7	30.4	33.3	38.5	47.6	65.6	106	220
6	10.53	32.0	31.5	31.9	33.2	35.8	40.4	48.4	63.5	95.1	173	425
7	11.77	35.3	35.1	35.9	37.9	41.8	48.5	60.7	84.7	138	286	829
8	12.99	38.7	38.8	40.1	43.1	48.5	58.1	76.1	113	203	476	1640
9	14.21	42.1	42.5	44.5	48.6	55.9	69.2	95.2	152	300	798	3240
10	15.41	45.6	46.4	49.2	54.6	64.3	82.5	119	205	446	1350	6470
11	16.60	49.1	50.4	54.1	61.0	73.8	98.3	150	277	666	2280	13000
12	17.78	52.6	54.6	59.3	68.1	84.5	117	189	377	1000	3890	26100
13	18.96	56.2	58.8	64.7	75.8	96.7	139	238	514	1510	6650	52700
14	20.13	59.9	63.3	70.6	84.2	111	166	302	703	2280	11400	
15	21.29	63.6	67.9	76.7	93.4	126	199	383	965	3450	19600	
16	22.45	67.5	72.7	83.3	104	145	238	487	1330	5240	33800	
17	23.61	71.5	77.8	90.4	115	166	285	622	1840	7990	58500	
18	24.76	75.4	82.8	97.7	127	190	342	794	2540	12200		
19	25.90	79.6	88.3	106	141	217	412	1020	3510	18600		
20	27.05	83.9	94.1	114	156	250	496	1310	4880	28600		
22	29.32	92.6	106	133	191	329	722	2160	9450	67400		
24	31.58	102	119	155	235	435	1060	3590	18400			
26	33.84	112	134	180	288	578	1560	6000	35900			
28	36.08	122	150	210	355	772	2300	10100				
30	38.32	133	168	245	439	1040	3420	17000				
35	43.87	164	221	358	750	2170	9300	63300				
40	49.39	200	290	529	1300	4650	25700					
45	54.88	243	381	784	2280	10000						
50	60.34	295	504	1180	4050	21900						
60	71.20	431	887	2690	13000							
70	81.99	632	1580	6320	42900							
80	92.78	932	2880	15100								
90	103.4	1390	5300	36400								
99	113.0	2000	9280									

c	r / m	0.15 M	0.10 M	0.05 M
0	2.303	10.3	12.5	19.2
1	3.890	23.6	37.7	106
2	5.322	52.8	117	629
3	6.681	121	382	3900
4	7.994	285	1280	24800
5	9.275	684	4380	159000
6	10.53	1670	15100	
7	11.77	4100	52700	
8	12.99	10200		
9	14.21	25400		
10	15.41	63900		

Single Sampling Tables with Consumers Risk of 10 %

$B(c,m) = 0.10$, $r = p_1/p_2$, $m = np_2$, $M = Np_2$, $\gamma = 5$.

	r	0.70	0.65	0.60	0.55	0.50	0.45	0.40
c	m	M	M	M	M	M	M	M
0	2.303	3.89	3.89	3.89	3.89	3.89	3.89	3.89
1	3.890	5.32	5.32	5.32	5.32	5.32	5.32	5.32
2	5.322	6.68	6.68	6.68	6.68	6.68	6.68	6.68
3	6.681	7.99	7.99	7.99	7.99	7.99	7.99	7.99
4	7.994	9.27	9.27	9.27	9.27	9.27	9.27	9.27
5	9.275	10.5	10.5	10.5	10.5	10.5	10.5	11.5
6	10.53	11.8	11.8	11.8	11.8	11.8	11.8	14.6
7	11.77	13.0	13.0	13.0	13.0	13.0	14.4	18.1
8	12.99	14.2	14.2	14.2	14.2	14.2	17.4	22.2
9	14.21	15.4	15.4	15.4	15.4	16.4	20.7	27.0
10	15.41	16.6	16.6	16.6	16.6	19.1	24.3	32.9
11	16.60	17.8	17.8	17.8	17.8	22.0	28.5	40.0
12	17.78	19.0	19.0	19.0	19.7	25.2	33.2	48.7
13	18.96	20.1	20.1	20.1	22.2	28.6	38.7	59.6
14	20.13	21.3	21.3	21.3	24.9	32.4	45.1	73.3
15	21.29	22.5	22.5	22.5	27.8	36.5	52.5	90.4
16	22.45	23.6	23.6	23.6	30.8	41.1	61.3	112
17	23.61	24.8	24.8	26.0	34.0	46.3	71.7	140
18	24.76	25.9	25.9	28.5	37.4	52.0	84.0	175
19	25.90	27.0	27.0	31.0	41.1	58.5	98.9	221
20	27.05	28.2	28.2	33.7	45.1	65.9	117	280
22	29.32	30.5	30.5	39.5	54.1	83.7	164	452
24	31.58	32.7	34.0	45.8	64.6	107	233	740
26	33.84	35.0	38.8	52.7	77.2	137	334	1220
28	36.08	37.2	44.0	60.5	92.5	178	485	2040
30	38.32	39.4	49.4	69.3	111	232	711	3420
35	43.87	46.2	64.8	96.7	178	465	1890	12700
40	49.39	58.2	83.3	135	293	964	5170	47900
45	54.88	71.4	106	191	493	2050	14400	
50	60.34	86.4	135	274	851	4430	40500	
60	71.20	123	221	586	2660	21500		
70	81.99	172	369	1320	8640			
80	92.73	241	636	3080	28800			
90	103.4	341	1130	7360				
99	113.0	472	1930	16300				

c	r m	0.35 M	0.30 M	0.25 M	0.20 M	0.15 M	0.10 M	0.05 M
0	2.303	3.89	3.89	3.89	3.89	3.89	3.89	5.46
1	3.890	5.32	5.32	5.32	5.59	7.05	10.2	24.0
2	5.322	6.68	6.68	7.79	9.87	14.1	27.3	130
3	6.681	7.99	9.53	12.0	16.7	28.8	81.4	786
4	7.994	10.8	13.4	18.1	28.5	62.6	263	4960
5	9.275	14.3	18.6	27.2	50.5	144	883	31900
6	10.53	18.5	25.5	41.7	92.5	341	3030	207000
7	11.77	23.8	35.2	65.2	174	830	10500	
8	12.99	30.5	49.2	104	337	2050	37000	
9	14.21	39.3	69.5	170	659	5100	131000	
10	15.41	50.8	99.7	280	1310	12800		
11	16.60	66.3	145	469	2610	32200		
12	17.78	87.2	212	791	5230	81900		
13	18.96	116	314	1340	10500			
14	20.13	154	470	2300	21300			
15	21.29	208	705	3940	43300			
16	22.45	281	1060	6780	88100			
17	23.61	384	1620	11700				
18	24.76	525	2460	20200				
19	25.90	721	3750	35100				
20	27.05	996	5740	61200				
22	29.32	1920	13500					
24	31.58	3700	31900					
26	33.84	7210	76700					
28	36.08	14100						
30	38.32	27700						

AQL single sampling tables with producer's risk of 5 % and minimum average costs.

The tables on pp. 19 - 28 are based on a binomial producer's risk of 5 %, $Q(p_1) = 0.05$, and a binomial consumer's risk, $P(p_2)$. The sampling plans given minimize the average costs $R_o = n + (N - n)\gamma P(p_2)$.

The same plans minimize the average costs $R = n + (N - n)(\gamma_1 Q(p_1) + \gamma_2 P(p_2))$ for $Q(p_1) = 0.05$ since $R = (1 - 0.05\gamma_1)R_o + 0.05\gamma_1 N$ with $\gamma = \gamma_2/(1 - 0.05\gamma_1)$.

The condition $Q(p_1) = 0.05$ has been fulfilled as nearly as possible in the way that n has been determined as the largest integer satisfying $B(c,n,p_1) \geqq 0.95$.

The tables give n, c and $100P(p_2)$ as functions of N for $\gamma = 0.2$ and 1.0, and for the following 50 combinations of $100p_1$ and $100p_2$:

$100p_1$	$100p_2$				
0.1	0.2	0.3	0.4	0.6	1.0
0.2	0.4	0.6	0.8	1.2	2.0
0.5	1.0	1.5	2.0	3.0	5.0
1.0	2.0	2.5	3.0	4.0	6.0
2.0	4.0	5.0	6.0	8.0	12.0
3.0	5.0	6.0	7.5	9.0	12.0
4.0	6.0	7.0	8.0	10.0	12.0
5.0	7.5	8.5	10.0	12.5	15.0
7.0	10.5	12.0	14.0	17.5	21.0
10.0	15.0	17.0	20.0	25.0	30.0

Methods of interpolation have been discussed in section 6.

The tables may be used for $\gamma \neq 0.2$ and $\gamma \neq 1.0$ in the following way: For $\gamma \leq 0.6$ compute $N^* = N\gamma/0.2$ and use the plan for N^* and $\gamma = 0.2$. For $0.6 < \gamma < 2$ compute $N^* = N\gamma$ and use the plan for N^* and $\gamma = 1$.

For $\gamma < 1$ it may happen that acceptance without inspection is cheaper than sampling inspection for small lots. In such cases the letter a has been added after the sample size.

The tables on pp. 29 - 30 are based on the same assumptions with the only modification that the consumer's and the producer's risks have been computed from Poisson probabilities. The functions $m = np_1$ and $M = Np_1$ have been tabulated for $M < 50{,}000$ with c and $r = p_2/p_1$ as arguments for $c \leq 99$ and r = 1.50, 1.60, 1.80, 2.00, 2.25, 2.50, 2.75, 3.0, 3.5, 4.0, 5.0, 6.5, 10.0, and for $\gamma = 0.2$ and 1.0. The optimum plan is (c,m) for $M(c-1) < M < M(c)$.

For $\gamma \leq 0.6$ use $M^* = M\gamma/0.2$ and the table for $\gamma = 0.2$. For $0.6 < \gamma < 2$ use $M^* = M\gamma$ and the table for $\gamma = 1$.

Underlining of M in the table for $\gamma = 0.2$ means that acceptance without inspection is cheaper than sampling inspection.

An approximation to the "binomial solution" may be obtained by using c from the Poisson table and correcting the corresponding n to $n_b = n + (c - np_1)/2$.

Single Sampling Tables for AQL = 0.1 per cent and γ = .2

$100p_2$	1.00			0.60			0.40			0.30			0.20		
N	n	c	100P	n	c	100P	n	c	100P	n	c	100P	n	c	100P
50	All	-	-	All	-	-	All	-	-	All	-	-	All	-	-
70	51a	0	59.9	51a	0	73.6	51a	0	81.5	51a	0	85.8	51a	0	90.3
100	51a	0	59.9	51a	0	73.6	51a	0	81.5	51a	0	85.8	51a	0	90.3
200	51a	0	59.9	51a	0	73.6	51a	0	81.5	51a	0	85.8	51a	0	90.3
300	51a	0	59.9	51a	0	73.6	51a	0	81.5	51a	0	85.8	51a	0	90.3
500	51a	0	59.9	51a	0	73.6	51a	0	81.5	51a	0	85.8	51a	0	90.3
700	51	0	59.9	51a	0	73.6	51a	0	81.5	51a	0	85.8	51a	0	90.3
1000	51	0	59.9	51	0	73.6	51a	0	81.5	51a	0	85.8	51a	0	90.3
2000	51	0	59.9	51	0	73.6	51	0	81.5	51	0	85.8	51a	0	90.3
3000	51	0	59.9	51	0	73.6	51	0	81.5	51	0	85.8	51	0	90.3
5000	355	1	12.9	355	1	37.1	51	0	81.5	51	0	85.8	51	0	90.3
7000	355	1	12.9	355	1	37.1	355	1	58.5	51	0	85.8	51	0	90.3
10000	355	1	12.9	818	2	13.2	355	1	58.5	355	1	71.2	51	0	90.3
20000	818	2	1.2	818	2	13.2	1367	3	20.5	1367	3	41.4	51	0	90.3
30000	818	2	1.2	1367	3	3.7	1367	3	20.5	1971	4	29.6	818	2	77.4
50000	818	2	1.2	1367	3	3.7	1971	4	10.6	2614	5	20.6	2614	5	57.6
70000	818	2	1.2	1367	3	3.7	2614	5	5.1	3286	6	13.9	4696	8	40.5
100000	818	2	1.2	1367	3	3.7	2614	5	5.1	3982	7	9.2	6926	11	27.2
200000	818	2	1.2	1971	4	0.8	3285	6	2.4	5427	9	3.7	10834	16	13.1

Single Sampling Tables for AQL = 0.2 per cent and γ = .2

$100p_2$	2.00			1.20			0.80			0.60			0.40		
N	n	c	100P	n	c	100P	n	c	100P	n	c	100P	n	c	100P
30	25a	0	60.3	25a	0	73.9	25a	0	81.8	25a	0	86.0	25a	0	90.5
50	25a	0	60.3	25a	0	73.9	25a	0	81.8	25a	0	86.0	25a	0	90.5
70	25a	0	60.3	25a	0	73.9	25a	0	81.8	25a	0	86.0	25a	0	90.5
100	25a	0	60.3	25a	0	73.9	25a	0	81.8	25a	0	86.0	25a	0	90.5
200	25a	0	60.3	25a	0	73.9	25a	0	81.8	25a	0	86.0	25a	0	90.5
300	25	0	60.3	25a	0	73.9	25a	0	81.8	25a	0	86.0	25a	0	90.5
500	25	0	60.3	25	0	73.9	25a	0	81.8	25a	0	86.0	25a	0	90.5
700	25	0	60.3	25	0	73.9	25	0	81.8	25a	0	86.0	25a	0	90.5
1000	25	0	60.3	25	0	73.9	25	0	81.8	25	0	86.0	25a	0	90.5
2000	178	1	12.7	178	1	36.9	25	0	81.8	25	0	86.0	25	0	90.5
3000	178	1	12.7	178	1	36.9	178	1	58.3	25	0	86.0	25	0	90.5
5000	178	1	12.7	409	2	13.1	178	1	58.3	178	1	71.1	25	0	90.5
7000	178	1	12.7	409	2	13.1	409	2	36.4	409	2	55.5	25	0	90.5
10000	178	1	12.7	409	2	13.1	683	3	20.5	683	3	41.4	178	1	84.0
20000	409	2	1.1	683	3	3.6	986	4	10.5	1307	5	20.5	683	3	70.7
30000	409	2	1.1	683	3	3.6	986	4	10.5	1644	6	13.8	1991	7	45.8
50000	409	2	1.1	683	3	3.6	1307	5	5.1	1991	7	9.1	3464	11	27.2
70000	409	2	1.1	986	4	0.8	1644	6	2.3	2349	8	5.9	4234	13	20.5
100000	409	2	1.1	986	4	0.8	1644	6	2.3	2714	9	3.7	5418	16	13.0
200000	683	3	0.1	986	4	0.8	1991	7	1.0	3086	10	2.3	7449	21	5.8

Single Sampling Tables for AQL = 0.5 per cent and γ = .2

$100p_2$	5.00			3.00			2.00			1.50			1.00		
N	n	c	100P	n	c	100P	n	c	100P	n	c	100P	n	c	100P
30	10a	0	59.9	10a	0	73.7	10a	0	81.7	10a	0	86.0	10a	0	90.4
50	10a	0	59.9	10a	0	73.7	10a	0	81.7	10a	0	86.0	10a	0	90.4
70	10a	0	59.9	10a	0	73.7	10a	0	81.7	10a	0	86.0	10a	0	90.4
100	10a	0	59.9	10a	0	73.7	10a	0	81.7	10a	0	86.0	10a	0	90.4
200	10	0	59.9	10	0	73.7	10a	0	81.7	10a	0	86.0	10a	0	90.4
300	10	0	59.9	10	0	73.7	10	0	81.7	10	0	86.0	10a	0	90.4
500	10	0	59.9	10	0	73.7	10	0	81.7	10	0	86.0	10	0	90.4
700	71	1	12.4	10	0	73.7	10	0	81.7	10	0	86.0	10	0	90.4
1000	71	1	12.4	71	1	36.8	10	0	81.7	10	0	86.0	10	0	90.4
2000	71	1	12.4	164	2	12.8	71	1	58.3	71	1	71.2	10	0	90.4
3000	71	1	12.4	164	2	12.8	164	2	36.1	164	2	55.3	10	0	90.4
5000	164	2	1.0	164	2	12.8	274	3	20.1	274	3	41.1	71	1	84.1
7000	164	2	1.0	274	3	3.5	395	4	10.3	395	4	29.3	164	2	77.3
10000	164	2	1.0	274	3	3.5	395	4	10.3	523	5	20.4	523	5	57.5
20000	164	2	1.0	274	3	3.5	523	5	5.0	797	7	9.0	1386	11	27.1
30000	164	2	1.0	395	4	0.8	658	6	2.3	940	8	5.8	1851	14	17.5
50000	164	2	1.0	395	4	0.8	658	6	2.3	1086	9	3.6	2329	17	11.0
70000	274	3	0.0	395	4	0.8	797	7	1.0	1235	10	2.3	2817	20	6.9
100000	274	3	0.0	395	4	0.8	797	7	1.0	1386	11	1.4	3147	22	4.8
200000	274	3	0.0	523	5	0.2	940	8	0.4	1540	12	0.8	3815	26	2.4

Single Sampling Tables for AQL = 1.0 per cent and γ = .2

$100p_2$	6.00			4.00			3.00			2.50			2.00		
N	n	c	100P	n	c	100P	n	c	100P	n	c	100P	n	c	100P
30	5a	0	73.4	5a	0	81.5	5a	0	85.9	5a	0	88.1	5a	0	90.4
50	5a	0	73.4	5a	0	81.5	5a	0	85.9	5a	0	88.1	5a	0	90.4
70	5a	0	73.4	5a	0	81.5	5a	0	85.9	5a	0	88.1	5a	0	90.4
100	5	0	73.4	5a	0	81.5	5a	0	85.9	5a	0	88.1	5a	0	90.4
200	5	0	73.4	5	0	81.5	5	0	85.9	5	0	88.1	5a	0	90.4
300	5	0	73.4	5	0	81.5	5	0	85.9	5	0	88.1	5	0	90.4
500	35	1	37.1	5	0	81.5	5	0	85.9	5	0	88.1	5	0	90.4
700	35	1	37.1	35	1	58.9	5	0	85.9	5	0	88.1	5	0	90.4
1000	82	2	12.4	82	2	35.8	35	1	71.7	5	0	88.1	5	0	90.4
2000	82	2	12.4	137	3	19.8	137	3	40.9	82	2	66.3	5	0	90.4
3000	82	2	12.4	137	3	19.8	198	4	28.9	198	4	44.7	82	2	77.4
5000	137	3	3.3	198	4	10.0	262	5	20.0	329	6	28.3	262	5	57.4
7000	137	3	3.3	262	5	4.8	329	6	13.5	399	7	21.9	471	8	40.0
10000	137	3	3.3	262	5	4.8	399	7	8.8	544	9	12.7	694	11	26.8
20000	198	4	0.7	329	6	2.2	544	9	3.5	694	11	7.0	1085	16	12.7
30000	198	4	0.7	399	7	0.9	618	10	2.2	848	13	3.8	1328	19	7.8
50000	198	4	0.7	399	7	0.9	694	11	1.3	927	14	2.7	1575	22	4.7
70000	262	5	0.1	471	8	0.4	694	11	1.3	1006	15	2.0	1741	24	3.3
100000	262	5	0.1	471	8	0.4	771	12	0.8	1166	17	1.0	1909	26	2.3
200000	262	5	0.1	544	9	0.1	848	13	0.5	1328	19	0.5	2248	30	1.1

Single Sampling Tables for AQL = 2.0 per cent and γ = .2

$100p_2$	12.00			8.00			6.00			5.00			4.00		
N	n	c	100P	n	c	100P	n	c	100P	n	c	100P	n	c	100P
30	2a	0	77.4	2a	0	84.6	2a	0	88.4	2a	0	90.2	2a	0	92.2
50	2	0	77.4	2a	0	84.6	2a	0	88.4	2a	0	90.2	2a	0	92.2
70	2	0	77.4	2	0	84.6	2a	0	88.4	2a	0	90.2	2a	0	92.2
100	2	0	77.4	2	0	84.6	2	0	88.4	2	0	90.2	2a	0	92.2
200	18	1	34.6	2	0	84.6	2	0	88.4	2	0	90.2	2	0	92.2
300	18	1	34.6	18	1	57.2	2	0	88.4	2	0	90.2	2	0	92.2
500	18	1	34.6	18	1	57.2	18	1	70.6	2	0	90.2	2	0	92.2
700	41	2	11.6	41	2	35.3	41	2	55.0	18	1	77.4	2	0	92.2
1000	41	2	11.6	69	3	18.8	69	3	40.0	41	2	66.3	18	1	83.9
2000	69	3	2.8	99	4	9.5	131	5	19.6	131	5	35.6	69	3	70.2
3000	69	3	2.8	99	4	9.5	165	6	12.9	200	7	21.3	200	7	45.0
5000	69	3	2.8	131	5	4.5	200	7	8.3	273	9	12.1	348	11	26.2
7000	99	4	0.6	165	6	1.9	236	8	5.2	310	10	9.0	425	13	19.6
10000	99	4	0.6	165	6	1.9	273	9	3.2	348	11	6.6	544	16	12.2
20000	99	4	0.6	200	7	0.8	310	10	2.0	425	13	3.5	706	20	6.3
30000	99	4	0.6	200	7	0.8	348	11	1.2	504	15	1.8	789	22	4.4
50000	131	5	0.1	236	8	0.3	386	12	0.7	544	16	1.3	956	26	2.2
70000	131	5	0.1	236	8	0.3	425	13	0.4	584	17	0.9	998	27	1.8
100000	131	5	0.1	273	9	0.1	425	13	0.4	624	18	0.6	1126	30	1.0
200000	165	6	0.0	273	9	0.1	504	15	0.1	706	20	0.3	1254	33	0.6

Single Sampling Tables for AQL = 3.0 per cent and γ = .2

$100p_2$	12.00			9.00			7.50			6.00			5.00		
N	n	c	100P	n	c	100P	n	c	100P	n	c	100P	n	c	100P
30	1a	0	88.0	1a	0	91.0	1a	0	92.5	1a	0	94.0	1a	0	95.0
50	1	0	88.0	1	0	91.0	1a	0	92.5	1a	0	94.0	1a	0	95.0
70	1	0	88.0	1	0	91.0	1	0	92.5	1	0	94.0	1a	0	95.0
100	1	0	88.0	1	0	91.0	1	0	92.5	1	0	94.0	1	0	95.0
200	12	1	56.9	1	0	91.0	1	0	92.5	1	0	94.0	1	0	95.0
300	12	1	56.9	12	1	70.5	1	0	92.5	1	0	94.0	1	0	95.0
500	27	2	35.5	27	2	55.7	12	1	77.4	12	1	84.0	1	0	95.0
700	46	3	18.2	46	3	39.6	46	3	54.4	12	1	84.0	12	1	88.2
1000	46	3	18.2	66	4	28.1	66	4	44.3	12	1	84.0	12	1	88.2
2000	66	4	9.0	110	6	12.5	134	7	20.5	134	7	44.3	46	3	80.3
3000	88	5	3.9	134	7	7.7	158	8	15.5	207	10	29.7	158	8	60.7
5000	88	5	3.9	158	8	4.8	207	10	8.7	310	14	16.3	337	15	38.1
7000	110	6	1.7	182	9	3.0	232	11	6.4	337	15	13.8	472	20	26.2
10000	110	6	1.7	182	9	3.0	258	12	4.6	417	18	8.5	610	25	17.7
20000	134	7	0.7	232	11	1.1	337	15	1.6	527	22	4.2	866	34	8.1
30000	158	8	0.3	258	12	0.6	363	16	1.2	610	25	2.4	1040	40	4.7
50000	158	8	0.3	284	13	0.4	390	17	0.8	695	28	1.4	1186	45	2.9
70000	158	8	0.3	284	13	0.4	417	18	0.6	723	29	1.1	1274	48	2.2
100000	182	9	0.1	310	14	0.2	444	19	0.4	780	31	0.8	1392	52	1.5
200000	207	10	0.0	337	15	0.1	499	21	0.2	895	35	0.3	1600	59	0.7

Single Sampling Tables for AQL = 4.0 per cent and γ = .2

$100p_2$	12.00			10.00			8.00			7.00			6.00		
N	n	c	100P	n	c	100P	n	c	100P	n	c	100P	n	c	100P
30	1a	0	88.0	1a	0	90.0	1a	0	92.0	1a	0	93.0	1a	0	94.0
50	1	0	88.0	1	0	90.0	1a	0	92.0	1a	0	93.0	1a	0	94.0
70	1	0	88.0	1	0	90.0	1	0	92.0	1	0	93.0	1	0	94.0
100	1	0	88.0	1	0	90.0	1	0	92.0	1	0	93.0	1	0	94.0
200	9	1	70.5	1	0	90.0	1	0	92.0	1	0	93.0	1	0	94.0
300	9	1	70.5	9	1	77.5	1	0	92.0	1	0	93.0	1	0	94.0
500	34	3	40.5	21	2	64.8	9	1	84.2	1	0	93.0	1	0	94.0
700	50	4	26.8	50	4	43.1	21	2	76.6	9	1	87.3	1	0	94.0
1000	66	5	18.1	66	5	34.3	50	4	62.9	21	2	82.1	9	1	90.2
2000	83	6	11.7	119	8	14.8	137	9	33.6	119	8	54.5	21	2	87.2
3000	101	7	7.1	137	9	11.2	194	12	21.6	194	12	39.5	101	7	74.0
5000	119	8	4.4	175	11	5.9	253	15	13.4	334	19	20.5	334	19	46.4
7000	137	9	2.7	194	12	4.3	293	17	9.6	396	22	15.1	501	27	32.3
10000	156	10	1.6	213	13	3.1	334	19	6.8	501	27	8.9	694	36	20.8
20000	175	11	0.9	253	15	1.5	437	24	2.8	629	33	4.6	979	49	10.5
30000	194	12	0.5	273	16	1.1	501	27	1.5	694	36	3.2	1157	57	6.7
50000	213	13	0.3	313	18	0.5	543	29	1.1	803	41	1.8	1359	66	4.0
70000	213	13	0.3	334	19	0.3	586	31	0.7	868	44	1.2	1472	71	2.9
100000	233	14	0.2	354	20	0.2	629	33	0.5	957	48	0.7	1608	77	2.0
200000	253	15	0.1	396	22	0.1	694	36	0.3	1045	52	0.5	1904	90	0.9

Single Sampling Tables for AQL = 5.0 per cent and γ = .2

$100p_2$	15.00			12.50			10.00			8.50			7.50		
N	n	c	100P	n	c	100P	n	c	100P	n	c	100P	n	c	100P
30	1	0	85.0	1a	0	87.5	1a	0	90.0	1a	0	91.5	1a	0	92.5
50	1	0	85.0	1	0	87.5	1	0	90.0	1	0	91.5	1a	0	92.5
70	1	0	85.0	1	0	87.5	1	0	90.0	1	0	91.5	1	0	92.5
100	1	0	85.0	1	0	87.5	1	0	90.0	1	0	91.5	1	0	92.5
200	7	1	71.7	1	0	87.5	1	0	90.0	1	0	91.5	1	0	92.5
300	16	2	56.1	7	1	78.5	1	0	90.0	1	0	91.5	1	0	92.5
500	28	3	37.7	28	3	52.9	1	0	90.0	1	0	91.5	1	0	92.5
700	40	4	26.3	53	5	33.6	28	3	69.5	1	0	91.5	1	0	92.5
1000	53	5	17.4	67	6	25.2	67	6	49.0	28	3	78.9	1	0	92.5
2000	81	7	6.7	110	9	10.6	125	10	28.4	125	10	50.2	28	3	84.6
3000	95	8	4.2	125	10	7.7	187	14	15.2	203	15	34.0	187	14	56.9
5000	110	9	2.4	140	11	5.6	235	17	9.2	334	23	16.9	351	24	36.5
7000	110	9	2.4	171	13	2.8	268	19	6.4	384	26	12.9	487	32	24.8
10000	125	10	1.4	187	14	2.0	284	20	5.4	452	30	8.7	608	39	17.4
20000	140	11	0.8	219	16	0.9	351	24	2.5	556	36	4.7	873	54	7.6
30000	155	12	0.5	235	17	0.7	401	27	1.4	643	41	2.8	998	61	5.1
50000	171	13	0.3	251	18	0.5	452	30	0.8	731	46	1.6	1197	72	2.6
70000	187	14	0.1	268	19	0.3	487	32	0.5	784	49	1.2	1251	75	2.2
100000	187	14	0.1	284	20	0.2	504	33	0.4	855	53	0.7	1397	83	1.3
200000	203	15	0.1	317	22	0.1	556	36	0.2	962	59	0.4	1580	93	0.7

Single Sampling Tables for AQL = 7.0 per cent and γ = .2

$100p_2$	21.00			17.50			14.00			12.00			10.50		
N	n	c	100P	n	c	100P	n	c	100P	n	c	100P	n	c	100P
30	5a	1	71.7	5a	1	78.8	5a	1	85.3	5a	1	88.8	5a	1	91.1
50	5a	1	71.7	5a	1	78.8	5a	1	85.3	5a	1	88.8	5a	1	91.1
70	5a	1	71.7	5a	1	78.8	5a	1	85.3	5a	1	88.8	5a	1	91.1
100	5	1	71.7	5	1	78.8	5a	1	85.3	5a	1	88.8	5a	1	91.1
200	12	2	52.3	5	1	78.8	5	1	85.3	5	1	88.8	5a	1	91.1
300	20	3	36.9	12	2	64.8	5	1	85.3	5	1	88.8	5	1	91.1
500	29	4	24.1	29	4	40.9	20	3	69.6	5	1	88.8	5	1	91.1
700	38	5	16.1	48	6	24.2	48	6	48.4	12	2	83.3	5	1	91.1
1000	48	6	9.7	58	7	18.1	79	9	31.7	48	6	64.8	12	2	87.6
2000	58	7	5.9	90	10	6.7	123	13	16.7	134	14	34.8	101	11	62.9
3000	68	8	3.6	101	11	4.7	157	16	10.0	204	20	19.7	204	20	42.8
5000	79	9	2.0	112	12	3.3	192	19	5.8	263	25	12.3	349	32	23.8
7000	90	10	1.1	123	13	2.3	204	20	4.7	300	28	8.8	423	38	17.5
10000	90	10	1.1	134	14	1.7	227	22	3.3	349	32	5.7	511	45	11.8
20000	101	11	0.6	157	16	0.8	275	26	1.5	423	38	3.0	689	59	5.2
30000	112	12	0.3	168	17	0.5	300	28	1.0	473	42	1.9	779	66	3.4
50000	123	13	0.2	192	19	0.2	324	30	0.6	536	47	1.0	882	74	2.1
70000	134	14	0.1	192	19	0.2	349	32	0.4	574	50	0.7	947	79	1.5
100000	13[illegible]	14	0.1	204	20	0.1	361	33	0.3	612	53	0.5	1026	85	1.0
200000	145	15	0.1	227	22	0.1	423	38	0.1	689	59	0.2	1157	95	0.5

Single Sampling Tables for AQL = 10.0 per cent and γ = .2

$100p_2$	30.00			25.00			20.00			17.00			15.00		
N	n	c	100P	n	c	100P	n	c	100P	n	c	100P	n	c	100P
30	3a	1	78.4	3a	1	84.4	3a	1	89.6	3a	1	92.3	3a	1	93.9
50	3a	1	78.4	3a	1	84.4	3a	1	89.6	3a	1	92.3	3a	1	93.9
70	3	1	78.4	3a	1	84.4	3a	1	89.6	3a	1	92.3	3a	1	93.9
100	8	2	55.2	3	1	84.4	3a	1	89.6	3a	1	92.3	3a	1	93.9
200	14	3	35.5	14	3	52.1	3	1	89.6	3	1	92.3	3a	1	93.9
300	20	4	23.8	20	4	41.5	14	3	69.8	3	1	92.3	3	1	93.9
500	27	5	13.6	34	6	21.8	34	6	46.6	14	3	79.6	8	2	89.5
700	34	6	7.9	41	7	16.1	56	9	29.3	34	6	64.6	14	3	85.3
1000	34	6	7.9	48	8	11.9	71	11	21.5	56	9	51.3	34	6	75.9
2000	48	8	2.7	63	10	5.8	102	15	11.0	127	18	23.7	152	21	39.3
3000	56	9	1.3	71	11	3.8	119	17	7.0	177	24	13.0	211	28	27.7
5000	56	9	1.3	79	12	2.5	152	21	3.1	211	28	8.5	289	37	16.8
7000	63	10	0.8	94	14	1.3	152	21	3.1	254	33	4.9	368	46	10.0
10000	63	10	0.8	102	15	0.8	177	24	1.7	280	35	3.5	421	52	7.0
20000	71	11	0.4	119	17	0.3	194	26	1.1	324	41	1.9	511	62	3.7
30000	79	12	0.2	119	17	0.3	211	28	0.7	359	45	1.2	574	69	2.4
50000	86	13	0.1	127	18	0.2	228	30	0.5	403	50	0.7	638	76	1.5
70000	86	13	0.1	135	19	0.1	254	33	0.2	421	52	0.5	702	83	0.9
100000	94	14	0.1	143	20	0.1	254	33	0.2	448	55	0.4	757	89	0.6
200000	102	15	0.0	152	21	0.1	289	37	0.1	502	61	0.2	[illegible]	[illegible]	[illegible]

Single Sampling Tables for AQL = 0.1 per cent and γ =1

$100p_2$	1.00			0.60			0.40			0.30			0.20		
N	n	c	100P	n	c	100P	n	c	100P	n	c	100P	n	c	100P
50	All	-	-	All	-	-	All	-	-	All	-	-	All	-	-
70	51	0	59.9	51	0	73.6	51	0	81.5	51	0	85.8	51	0	90.3
100	51	0	59.9	51	0	73.6	51	0	81.5	51	0	85.8	51	0	90.3
200	51	0	59.9	51	0	73.6	51	0	81.5	51	0	85.8	51	0	90.3
300	51	0	59.9	51	0	73.6	51	0	81.5	51	0	85.8	51	0	90.3
500	51	0	59.9	51	0	73.6	51	0	81.5	51	0	85.8	51	0	90.3
700	355	1	12.9	355	1	37.1	355	1	58.5	355	1	71.2	51	0	90.3
1000	355	1	12.9	355	1	37.1	355	1	58.5	355	1	71.2	355	1	84.1
2000	355	1	12.9	355	1	37.1	818	2	36.5	818	2	55.5	818	2	77.4
3000	355	1	12.9	818	2	13.2	818	2	36.5	818	2	55.5	818	2	77.4
5000	818	2	1.2	818	2	13.2	1367	3	20.5	1971	4	29.6	1971	4	64.0
7000	818	2	1.2	1367	3	3.7	1971	4	10.6	1971	4	29.6	2614	5	57.6
10000	818	2	1.2	1367	3	3.7	1971	4	10.6	2614	5	20.6	3982	7	45.8
20000	818	2	1.2	1367	3	3.7	2614	5	5.1	3982	7	9.2	6926	11	27.2
30000	818	2	1.2	1971	4	0.8	3286	6	2.4	4696	8	5.9	8466	13	20.5
50000	818	2	1.2	1971	4	0.8	3286	6	2.4	5427	9	3.7	11637	17	11.2
70000	1367	3	0.1	1971	4	0.8	3982	7	1.0	6170	10	2.3	13257	19	8.1
100000	1367	3	0.1	2614	5	0.2	3982	7	1.0	6926	11	1.4	14896	21	5.8
200000	1367	3	0.1	2614	5	0.2	4696	8	0.4	7691	12	0.9	19061	26	2.5

Single Sampling Tables for AQL = 0.2 per cent and γ =1

$100p_2$	2.00			1.20			0.80			0.60			0.40		
N	n	c	100P	n	c	100P	n	c	100P	n	c	100P	n	c	100P
30	25	0	60.3	25	0	73.9	25	0	81.8	25	0	86.0	25	0	90.5
50	25	0	60.3	25	0	73.9	25	0	81.8	25	0	86.0	25	0	90.5
70	25	0	60.3	25	0	73.9	25	0	81.8	25	0	86.0	25	0	90.5
100	25	0	60.3	25	0	73.9	25	0	81.8	25	0	86.0	25	0	90.5
200	25	0	60.3	25	0	73.9	25	0	81.8	25	0	86.0	25	0	90.5
300	25	0	60.3	178	1	36.9	178	1	58.3	25	0	86.0	25	0	90.5
500	178	1	12.7	178	1	36.9	178	1	58.3	178	1	71.1	178	1	84.0
700	178	1	12.7	178	1	36.9	178	1	58.3	178	1	71.1	178	1	84.0
1000	178	1	12.7	178	1	36.9	409	2	36.4	409	2	55.5	409	2	77.4
2000	178	1	12.7	409	2	13.1	683	3	20.5	683	3	41.4	683	3	70.7
3000	409	2	1.1	409	2	13.1	683	3	20.5	986	4	29.6	986	4	64.0
5000	409	2	1.1	683	3	3.6	986	4	10.5	1307	5	20.5	1991	7	45.8
7000	409	2	1.1	683	3	3.6	1307	5	5.1	1644	6	13.8	2349	8	40.4
10000	409	2	1.1	683	3	3.6	1307	5	5.1	1991	7	9.1	3464	11	27.2
20000	409	2	1.1	986	4	0.8	1644	6	2.3	2714	9	3.7	5020	15	15.2
30000	683	3	0.1	986	4	0.8	1991	7	1.0	3086	10	2.3	6223	18	9.5
50000	683	3	0.1	1307	5	0.2	1991	7	1.0	3464	11	1.4	7449	21	5.8
70000	683	3	0.1	1307	5	0.2	2349	8	0.4	3464	11	1.4	8694	24	3.5
100000	683	3	0.1	1307	5	0.2	2349	8	0.4	3846	12	0.9	9532	26	2.4
200000	683	3	0.1	1307	5	0.2	2714	9	0.2	4625	14	0.3	11226	30	1.2

Single Sampling Tables for AQL = 0.5 per cent and γ =1

$100p_2$	5.00			3.00			2.00			1.50			1.00		
N	n	c	100P	n	c	100P	n	c	100P	n	c	100P	n	c	100P
30	10	0	59.9	10	0	73.7	10	0	81.7	10	0	86.0	10	0	90.4
50	10	0	59.9	10	0	73.7	10	0	81.7	10	0	86.0	10	0	90.4
70	10	0	59.9	10	0	73.7	10	0	81.7	10	0	86.0	10	0	90.4
100	10	0	59.9	10	0	73.7	10	0	81.7	10	0	86.0	10	0	90.4
200	71	1	12.4	71	1	36.8	71	1	58.3	71	1	71.2	71	1	84.1
300	71	1	12.4	71	1	36.8	71	1	58.3	71	1	71.2	71	1	84.1
500	71	1	12.4	164	2	12.8	164	2	36.1	164	2	55.3	164	2	77.3
700	71	1	12.4	164	2	12.8	164	2	36.1	274	3	41.1	274	3	70.6
1000	164	2	1.0	164	2	12.8	274	3	20.1	274	3	41.1	395	4	63.9
2000	164	2	1.0	274	3	3.5	395	4	10.3	523	5	20.4	797	7	45.6
3000	164	2	1.0	274	3	3.5	523	5	5.0	658	6	13.7	1086	9	35.5
5000	164	2	1.0	395	4	0.8	523	5	5.0	940	8	5.8	1540	12	23.5
7000	164	2	1.0	395	4	0.8	658	6	2.3	940	8	5.8	1851	14	17.5
10000	164	2	1.0	395	4	0.8	658	6	2.3	1086	9	3.6	2329	17	11.0
20000	274	3	0.0	395	4	0.8	797	7	1.0	1386	11	1.4	2981	21	5.7
30000	274	3	0.0	523	5	0.2	940	8	0.4	1540	12	0.8	3479	24	3.4
50000	274	3	0.0	523	5	0.2	940	8	0.4	1695	13	0.5	3983	27	2.0
70000	274	3	0.0	523	5	0.2	1086	9	0.2	1695	13	0.5	4322	29	1.4
100000	274	3	0.0	523	5	0.2	1086	9	0.2	1851	14	0.3	4663	31	1.0
200000	274	3	0.0	658	6	0.0	1235	10	0.1	2009	15	0.2	5350	35	0.5

Single Sampling Tables for AQL = 1.0 per cent and γ =1

$100p_2$	6.00			4.00			3.00			2.50			2.00		
N	n	c	100P	n	c	100P	n	c	100P	n	c	100P	n	c	100P
30	5	0	73.4	5	0	81.5	5	0	85.9	5	0	88.1	5	0	90.4
50	5	0	73.4	5	0	81.5	5	0	85.9	5	0	88.1	5	0	90.4
70	35	1	37.1	35	1	58.9	35	1	71.7	5	0	88.1	5	0	90.4
100	35	1	37.1	35	1	58.9	35	1	71.7	35	1	78.2	35	1	84.5
200	35	1	37.1	82	2	35.8	82	2	55.2	82	2	66.3	82	2	77.4
300	82	2	12.4	82	2	35.8	82	2	55.2	82	2	66.3	82	2	77.4
500	82	2	12.4	137	3	19.8	198	4	28.9	198	4	44.7	198	4	63.7
700	137	3	3.3	198	4	10.0	198	4	28.9	262	5	35.9	262	5	57.4
1000	137	3	3.3	198	4	10.0	262	5	20.0	329	6	28.3	399	7	45.4
2000	137	3	3.3	262	5	4.8	399	7	8.8	471	8	16.7	694	11	26.8
3000	198	4	0.7	329	6	2.2	471	8	5.6	618	10	9.5	848	13	20.1
5000	198	4	0.7	329	6	2.2	544	9	3.5	771	12	5.2	1166	17	10.8
7000	198	4	0.7	399	7	0.9	618	10	2.2	848	13	3.8	1328	19	7.8
10000	198	4	0.7	399	7	0.9	694	11	1.3	927	14	2.7	1492	21	5.6
20000	262	5	0.1	471	8	0.4	771	12	0.8	1085	16	1.4	1909	26	2.3
30000	262	5	0.1	471	8	0.4	848	13	0.5	1246	18	0.7	2078	28	1.6
50000	262	5	0.1	544	9	0.1	927	14	0.3	1328	19	0.5	2333	31	0.9
70000	329	6	0.0	544	9	0.1	927	14	0.3	1410	20	0.3	2505	33	0.6
100000	329	6	0.0	618	10	0.1	1006	15	0.2	1492	21	0.2	2677	35	0.4
200000	329	6	0.0	618	10	0.1	1085	16	0.1	1658	23	0.1	2937	38	0.2

Single Sampling Tables for AQL = 2.0 per cent and γ = 1

$100p_2$	12.00			8.00			6.00			5.00			4.00		
N	n	c	100P	n	c	100P	n	c	100P	n	c	100P	n	c	100P
30	18	1	34.6	18	1	57.2	18	1	70.6	2	0	90.2	2	0	92.2
50	18	1	34.6	18	1	57.2	18	1	70.6	18	1	77.4	18	1	85.9
70	18	1	34.6	18	1	57.2	18	1	70.6	18	1	77.4	18	1	85.9
100	18	1	34.6	41	2	35.3	41	2	55.0	41	2	65.3	41	2	77.5
200	41	2	11.6	69	3	18.8	69	3	40.0	69	3	54.5	69	3	70.2
300	41	2	11.6	69	3	18.8	99	4	28.5	99	4	44.5	99	4	65.7
500	69	3	2.8	99	4	9.5	131	5	19.6	165	6	27.7	200	7	45.0
700	69	3	2.8	99	4	9.5	165	6	12.9	200	7	21.3	236	8	39.6
1000	69	3	2.8	131	5	4.5	200	7	8.3	236	8	16.2	310	10	30.2
2000	99	4	0.6	165	6	1.9	236	8	5.2	348	11	6.6	504	15	14.5
3000	99	4	0.6	165	6	1.9	273	9	3.2	386	12	4.9	624	18	8.9
5000	99	4	0.6	200	7	0.8	310	10	2.0	464	14	2.6	747	21	5.3
7000	131	5	0.1	200	7	0.8	348	11	1.2	504	15	1.8	830	23	3.7
10000	131	5	0.1	236	8	0.3	386	12	0.7	544	16	1.3	914	25	2.6
20000	131	5	0.1	273	9	0.1	425	13	0.4	624	18	0.6	1093	29	1.2
30000	131	5	0.1	273	9	0.1	464	14	0.2	665	19	0.4	1211	32	0.7
50000	165	6	0.0	273	9	0.1	504	15	0.1	747	21	0.2	1297	34	0.5
70000	165	6	0.0	310	10	0.0	544	16	0.1	789	22	0.1	1383	36	0.3
100000	165	6	0.0	310	10	0.0	544	16	0.1	830	23	0.1	1470	38	0.2
200000	165	6	0.0	348	11	0.0	584	17	0.0	914	25	0.0	1601	41	0.1

Single Sampling Tables for AQL = 3.0 per cent and γ = 1

$100p_2$	12.00			9.00			7.50			6.00			5.00		
N	n	c	100P	n	c	100P	n	c	100P	n	c	100P	n	c	100P
30	12	1	56.9	12	1	70.5	12	1	77.4	12	1	84.0	12	1	88.2
50	12	1	56.9	12	1	70.5	12	1	77.4	12	1	84.0	12	1	88.2
70	27	2	35.5	27	2	55.7	27	2	67.9	27	2	78.1	12	1	88.2
100	27	2	35.5	46	3	39.6	46	3	54.4	46	3	70.3	27	2	85.0
200	46	3	18.2	66	4	28.1	66	4	44.3	66	4	63.7	66	4	76.6
300	66	4	9.0	88	5	18.6	88	5	34.6	110	6	50.8	110	6	68.8
500	88	5	3.9	110	6	12.5	134	7	20.5	158	8	38.9	182	9	57.4
700	88	5	3.9	134	7	7.7	158	8	15.5	232	11	25.9	258	12	47.2
1000	88	5	3.9	158	8	4.8	207	10	8.7	284	13	19.0	337	15	38.1
2000	110	6	1.7	182	9	3.0	258	12	4.6	417	18	8.5	582	24	19.2
3000	134	7	0.7	207	10	1.8	284	13	3.3	472	20	5.9	723	29	12.6
5000	134	7	0.7	232	11	1.1	337	15	1.6	554	23	3.5	895	35	7.4
7000	158	8	0.3	258	12	0.6	363	16	1.2	610	25	2.4	1040	40	4.7
10000	158	8	0.3	258	12	0.6	390	17	0.8	695	28	1.4	1127	43	3.6
20000	182	9	0.1	310	14	0.2	444	19	0.4	780	31	0.8	1392	52	1.5
30000	182	9	0.1	337	15	0.1	472	20	0.3	837	33	0.5	1511	56	1.0
50000	207	10	0.0	337	15	0.1	527	22	0.1	924	36	0.3	1660	61	0.6
70000	207	10	0.0	363	16	0.1	527	22	0.1	981	38	0.2	1750	64	0.4
100000	207	10	0.0	390	17	0.0	582	24	0.1	1040	40	0.1	1870	68	0.3
200000	232	11	0.0	417	18	0.0	610	25	0.0	1127	43	0.1	2051	74	0.2

Single Sampling Tables for AQL = 4.0 per cent and γ =1

$100p_2$	12.00			10.00			8.00			7.00			6.00		
N	n	c	100P	n	c	100P	n	c	100P	n	c	100P	n	c	100P
30	9	1	70.5	9	1	77.5	9	1	84.2	9	1	87.3	9	1	90.2
50	21	2	53.0	21	2	64.8	21	2	76.6	9	1	87.3	9	1	90.2
70	21	2	53.0	21	2	64.8	21	2	76.6	21	2	82.1	21	2	87.2
100	34	3	40.5	34	3	55.4	34	3	71.2	21	2	82.1	21	2	87.2
200	50	4	26.8	66	5	34.3	83	6	50.0	83	6	63.8	66	5	79.6
300	83	6	11.7	101	7	19.7	101	7	43.6	119	8	54.5	101	7	74.0
500	101	7	7.1	119	8	14.8	156	10	28.9	175	11	42.8	175	11	64.1
700	101	7	7.1	137	9	11.2	194	12	21.6	233	14	33.2	273	16	52.7
1000	119	8	4.4	175	11	5.9	253	15	13.4	293	17	25.1	375	21	42.6
2000	156	10	1.6	213	13	3.1	334	19	6.8	458	25	11.2	629	33	24.2
3000	156	10	1.6	233	14	2.2	396	22	3.9	543	29	7.2	803	41	16.0
5000	175	11	0.9	273	16	1.1	458	25	2.3	629	33	4.6	1023	51	9.4
7000	194	12	0.5	293	17	0.7	501	27	1.5	694	36	3.2	1157	57	6.7
10000	213	13	0.3	313	18	0.5	522	28	1.3	781	40	2.0	1314	64	4.5
20000	233	14	0.2	354	20	0.2	629	33	0.5	912	46	1.0	1608	77	2.0
30000	253	15	0.1	375	21	0.2	672	35	0.3	1001	50	0.6	1790	85	1.3
50000	273	16	0.1	396	22	0.1	715	37	0.2	1090	54	0.3	1973	93	0.8
70000	273	16	0.1	416	23	0.1	759	39	0.1	1157	57	0.2	2088	98	0.5
100000	293	17	0.0	437	24	0.1	781	40	0.1	1224	60	0.2			
200000	313	18	0.0	479	26	0.0	866	44	0.0	1314	64	0.1			

Single Sampling Tables for AQL = 5.0 per cent and γ =1

$100p_2$	15.00			12.50			10.00			8.50			7.50		
N	n	c	100P	n	c	100P	n	c	100P	n	c	100P	n	c	100P
30	7	1	71.7	7	1	78.5	7	1	85.0	7	1	88.6	1	0	92.5
50	16	2	56.1	16	2	67.7	16	2	78.9	16	2	85.0	7	1	90.8
70	28	3	37.7	28	3	52.9	28	3	69.5	28	3	78.9	28	3	84.6
100	28	3	37.7	40	4	42.9	40	4	60.9	28	3	78.9	28	3	84.6
200	53	5	17.4	67	6	25.2	67	6	49.0	81	7	61.7	67	6	76.4
300	67	6	10.7	81	7	19.1	110	9	32.9	110	9	53.9	110	9	68.9
500	81	7	6.7	110	9	10.6	140	11	24.7	187	14	36.9	203	15	54.4
700	95	8	4.2	125	10	7.7	187	14	15.2	235	17	28.9	268	19	45.8
1000	110	9	2.4	140	11	5.6	219	16	10.9	284	20	22.3	351	24	36.5
2000	125	10	1.4	187	14	2.0	284	20	5.4	435	29	9.6	556	36	20.3
3000	140	11	0.8	203	15	1.4	334	23	3.1	504	33	6.4	696	44	13.3
5000	155	12	0.5	219	16	0.9	384	26	1.8	608	39	3.4	909	56	6.8
7000	155	12	0.5	235	17	0.7	418	28	1.2	643	41	2.8	1016	62	4.8
10000	171	13	0.3	251	18	0.5	435	29	1.0	731	46	1.6	1124	68	3.4
20000	187	14	0.1	284	20	0.2	504	33	0.4	837	52	0.8	1342	80	1.6
30000	203	15	0.1	300	21	0.1	538	35	0.3	909	56	0.5	1470	87	1.0
50000	219	16	0.0	317	22	0.1	573	37	0.2	998	61	0.3	1655	96	0.6
70000	219	16	0.0	334	23	0.1	608	39	0.1	1034	63	0.2			
100000	235	17	0.0	351	24	0.0	643	41	0.1	1088	66	0.2			
200000	251	18	0.0	384	26	0.0	696	44	0.0	1197	72	0.1			

Single Sampling Tables for AQL = 7.0 per cent and γ =1

$100p_2$	21.00			17.50			14.00			12.00			10.50		
N	n	c	100P	n	c	100P	n	c	100P	n	c	100P	n	c	100P
30	12	2	52.3	12	2	64.8	12	2	77.0	12	2	83.3	5	1	91.1
50	20	3	36.9	20	3	52.6	20	3	69.6	20	3	78.7	12	2	87.6
70	20	3	36.9	29	4	40.9	29	4	61.7	29	4	73.6	20	3	84.9
100	29	4	24.1	29	4	40.9	38	5	55.6	29	4	73.6	29	4	81.7
200	48	6	9.7	58	7	18.1	79	9	31.7	79	9	52.0	79	9	68.5
300	58	7	5.9	68	8	13.7	90	10	26.9	112	12	40.6	112	12	60.5
500	68	8	3.6	90	10	6.7	123	13	16.7	157	16	29.0	192	19	45.0
700	68	8	3.6	101	11	4.7	157	16	10.0	204	20	19.7	251	24	36.0
1000	79	9	2.0	112	12	3.3	180	18	7.0	251	24	13.6	312	29	27.9
2000	90	10	1.1	134	14	1.7	227	22	3.3	349	32	5.7	511	45	11.8
3000	101	11	0.6	145	15	1.2	251	24	2.2	386	35	4.1	587	51	8.4
5000	112	12	0.3	157	16	0.8	300	28	1.0	448	40	2.4	702	60	4.9
7000	112	12	0.3	180	18	0.3	300	28	1.0	498	44	1.5	792	67	3.2
10000	123	13	0.2	192	19	0.2	324	30	0.6	536	47	1.0	869	73	2.2
20000	134	14	0.1	204	20	0.1	361	33	0.3	612	53	0.5	1026	85	1.0
30000	145	15	0.1	215	21	0.1	386	35	0.2	638	55	0.4	1104	91	0.7
50000	157	16	0.0	227	22	0.1	423	38	0.1	702	60	0.2	1196	98	0.4
70000	157	16	0.0	239	23	0.0	448	40	0.1	740	63	0.1			
100000	168	17	0.0	251	24	0.0	473	42	0.0	779	66	0.1			
200000	180	18	0.0	275	26	0.0	511	45	0.0	843	71	0.1			

Single Sampling Tables for AQL = 10.0 per cent and γ =1

$100p_2$	30.00			25.00			20.00			17.00			15.00		
N	n	c	100P	n	c	100P	n	c	100P	n	c	100P	n	c	100P
30	14	3	35.5	14	3	52.1	14	3	69.8	14	3	79.6	14	3	85.3
50	14	3	35.5	20	4	41.5	20	4	63.0	14	3	79.6	14	3	85.3
70	20	4	23.8	27	5	29.9	27	5	53.9	27	5	69.5	27	5	79.0
100	27	5	13.6	34	6	21.8	34	6	46.6	34	6	64.6	34	6	75.9
200	34	6	7.9	48	8	11.9	56	9	29.3	79	12	40.2	79	12	59.5
300	41	7	4.6	56	9	7.8	79	12	17.8	119	17	25.8	119	17	47.6
500	48	8	2.7	71	11	3.8	110	16	9.1	152	21	17.5	177	24	34.1
700	56	9	1.3	79	12	2.5	119	17	7.0	177	24	13.0	228	30	25.0
1000	56	9	1.3	79	12	2.5	135	19	4.9	211	28	8.5	289	37	16.8
2000	63	10	0.8	102	15	0.8	177	24	1.7	254	33	4.9	377	47	9.4
3000	71	11	0.4	110	16	0.6	194	26	1.1	289	37	3.1	466	57	5.1
5000	79	12	0.2	119	17	0.3	211	28	0.7	350	44	1.4	538	65	3.1
7000	79	12	0.2	119	17	0.3	228	30	0.5	368	46	1.1	583	70	2.2
10000	86	13	0.1	127	18	0.2	228	30	0.5	377	47	0.9	638	76	1.5
20000	94	14	0.1	143	20	0.1	254	33	0.2	448	55	0.4	757	89	0.6
30000	102	15	0.0	152	21	0.1	280	36	0.1	475	58	0.2	812	95	0.4
50000	102	15	0.0	160	22	0.0	289	37	0.1	511	62	0.1	849	99	0.3
70000	110	16	0.0	168	23	0.0	315	40	0.0	538	65	0.1			
100000	110	16	0.0	177	24	0.0	324	41	0.0	565	68	0.1			
200000	119	17	0.0	185	25	0.0	350	44	0.0	629	75	0.0			

Single Sampling Tables with Producer's Risk of 5%.

$B(c,m) = 0.95$, $r = p_2/p_1$, $m = np_1$, $M = Np_1$, $\gamma = 0.2$.

	r	1.50	1.60	1.80	2.00	2.25	2.50	2.75	3.0	3.5	4.0	5.0
c	m	M	M	M	M	M	M	M	M	M	M	M
0	0.0513	47.8	38.3	27.0	20.5	15.6	12.6	10.5	9.01	7.08	5.90	4.59
1	0.3554	74.2	58.4	39.9	29.9	22.6	18.3	15.5	13.6	11.3	10.1	9.40
2	0.8177	89.5	69.8	47.5	35.8	27.6	23.0	20.2	18.6	17.1	17.3	20.8
3	1.366	101	78.6	53.7	41.1	32.8	28.5	26.4	25.6	26.9	31.2	52.4
4	1.970	109	85.1	59.0	46.3	38.6	35.3	34.7	35.9	43.6	59.9	148
5	2.613	116	91.0	64.3	52.1	45.5	44.1	46.2	51.4	73.5	122	465
6	3.285	123	96.8	70.1	58.6	54.0	55.6	62.5	75.3	129	263	1590
7	3.981	129	103	76.2	66.1	64.4	70.9	85.8	112	234	595	5810
8	4.695	134	108	82.6	74.5	76.9	90.9	119	171	438	1400	22500
9	5.425	141	114	90.0	84.5	92.8	118	169	266	844	3410	91800
10	6.169	146	120	98.0	96.0	112	154	241	421	1670	8590	
11	6.924	152	126	107	109	137	204	350	677	3360	22200	
12	7.690	158	133	116	125	167	271	514	1110	6920	58700	
13	8.464	164	140	127	143	206	365	762	1830	14500		
14	9.246	171	148	139	164	255	494	1140	3060	30900		
15	10.04	178	156	153	189	318	674	1730	5180	66600		
16	10.83	184	164	167	218	397	925	2640	8860			
17	11.63	192	174	184	253	499	1280	4070	15300			
18	12.44	199	184	203	293	630	1780	6300	26600			
19	13.25	207	194	223	341	797	2490	9840	46700			
20	14.07	216	205	247	398	1020	3510	15500	82700			
22	15.72	233	230	301	546	1660	7050	38800				
24	17.38	253	258	370	757	2760	14400					
26	19.06	274	290	458	1060	4650	30000					
28	20.75	298	327	569	1490	7930	63100					
30	22.44	324	369	710	2120	13600						
35	26.73	401	505	1260	5260	55100						
40	31.07	501	701	2300	13500							
45	35.44	631	984	4310	36000							
50	39.85	798	1400	8210								
60	48.75	1310	2910	31400								
70	57.73	2200	6300									
80	66.79	3780	14100									
90	75.90	6610	32300									
99	84.14	11100										

	r	6.5	10.0
c	m	M	M
0	0.0513	3.72	3.22
1	0.3554	10.3	19.9
2	0.8177	36.1	242
3	1.366	162	5040
4	1.970	890	153000
5	2.613	5730	
6	3.285	41800	
7	3.981	336000	

Single Sampling Tables with Producer's Risk of 5%.

$B(c,m) = 0.95$, $r = p_2/p_1$, $m = np_1$, $M = Np_1$, $\gamma = 1$.

c	r m	1.50 M	1.60 M	1.80 M	2.00 M	2.25 M	2.50 M	2.75 M	3.0 M	3.5 M	4.0 M	5.0 M
0	0.0513	1.22	1.09	0.932	0.837	0.762	0.713	0.679	0.655	0.622	0.602	0.583
1	0.3554	2.62	2.38	2.10	1.93	1.81	1.74	1.70	1.68	1.67	1.70	1.82
2	0.8177	4.06	3.76	3.39	3.20	3.08	3.04	3.04	3.09	3.26	3.54	4.52
3	1.366	5.60	5.23	4.83	4.64	4.58	4.63	4.77	4.99	5.69	6.82	11.3
4	1.970	7.14	6.74	6.34	6.21	6.28	6.54	6.97	7.59	9.56	13.1	31.0
5	2.613	8.73	8.33	7.97	7.96	8.26	8.88	9.85	11.3	16.1	26.1	94.9
6	3.285	10.4	9.98	9.71	9.88	10.6	11.8	13.7	16.6	27.7	54.8	320
7	3.981	12.1	11.7	11.6	12.0	13.3	15.4	19.0	24.6	49.3	122	1170
8	4.695	13.8	13.4	13.5	14.4	16.4	20.1	26.3	37.0	90.7	283	4510
9	5.425	15.6	15.3	15.7	17.0	20.2	26.1	36.8	56.6	173	687	18400
10	6.169	17.4	17.2	18.0	20.0	24.8	34.0	51.9	88.2	338	1720	78000
11	6.924	19.2	19.2	20.4	23.3	30.3	44.5	74.3	140	677	4440	
12	7.690	21.2	21.3	23.1	27.1	37.1	58.7	108	226	1390	11800	
13	8.464	23.1	23.5	25.9	31.4	45.5	78.0	158	372	2910	31800	
14	9.246	25.2	25.7	29.1	36.3	56.0	105	235	619	6180	87700	
15	10.04	27.3	28.1	32.4	42.0	69.2	141	353	1040	13300		
16	10.83	29.4	30.5	36.1	48.4	85.6	192	535	1780	29100		
17	11.63	31.6	33.1	40.2	56.0	107	264	821	3070	64500		
18	12.44	33.9	35.8	44.6	64.9	134	365	1270	5340			
19	13.25	36.2	38.6	49.4	75.1	168	508	1980	9360			
20	14.07	38.7	41.7	54.8	87.3	212	712	3100	16500			
22	15.72	43.8	48.0	67.1	118	343	1420	7780	52800			
24	17.38	49.2	55.1	82.4	162	564	2900	19900				
26	19.06	55.1	63.1	101	223	944	6010	52100				
28	20.75	61.3	71.9	125	312	1600	12600					
30	22.44	68.1	81.9	155	438	2740	27000					
35	26.73	87.4	113	269	1070	11000						
40	31.07	111	156	481	2730	46700						
45	35.44	141	216	885	7220							
50	39.85	178	302	1670	19700							
60	48.75	288	613	6320								
70	57.73	473	1300	25500								
80	66.79	796	2860									
90	75.90	1370	6510									
99	84.14	2280	14000									

c	r m	6.5 M	10.0 M
0	0.0513	0.579	0.617
1	0.3554	2.18	4.22
2	0.8177	7.74	49.1
3	1.366	33.4	1010
4	1.970	179	30600
5	2.613	1150	1230000
6	3.285	8360	
7	3.981	67300	

IQL single sampling tables with minimum average costs.

The tables on pp. 33 - 37 are based on a binomial risk of 50 % for lots of quality p_o, i.e. $P(p_o) = 0.50$, and a binomial producer's risk, $Q(p_1) = 1 - P(p_1)$. The sampling plans given minimize the average costs $R_o = n + (N - n)\gamma Q(p_1)$.

The same plans will with good approximation minimize the average costs $R = n + (N - n)(\gamma_1 Q(p_1) + \gamma_2 P(p_2))$ for $P(p_o) = 0.50$ where $\gamma = \gamma_1 + \gamma_2$ and $p_o = (\log \frac{q_1}{q_2}) / (\log \frac{p_2 q_1}{p_1 q_2})$.

The condition $P(p_o) = 0.50$ has been fulfilled as nearly as possible in the way that n has been determined as the integer for which $B(c,n,p_o)$ is nearest to 0.50.

The tables give n,c, and 100 $P(p_1)$ as functions of N for $\gamma = 1$ and for the following 45 combinations of 100 p_o and 100 p_1 (100 p_2 has been added in parenthesis after 100 p_1):

$100p_o$	$100p_1(100p_2)$				
0.5	0.1 (1.42)	0.15 (1.18)	0.2 (1.01)	0.25 (0.877)	0.3 (0.773)
1	0.2 (2.84)	0.3 (2.35)	0.4 (2.01)	0.5 (1.75)	0.6 (1.55)
2	0.4 (5.63)	0.6 (4.68)	0.8 (4.01)	1.0 (3.50)	1.2 (3.09)
3	0.6 (8.39)	0.9 (6.99)	1.2 (6.00)	1.5 (5.24)	1.8 (4.62)
4	1.2 (9.27)	1.6 (7.97)	2.0 (6.96)	2.4 (6.16)	2.8 (5.49)
5	1.5 (11.5)	2.0 (9.92)	2.5 (8.69)	3.0 (7.69)	3.5 (6.85)
7	2.8 (13.8)	3.5 (12.1)	4.2 (10.7)	4.9 (9.58)	5.6 (8.60)
10	4.0 (19.5)	5.0 (17.2)	6.0 (15.3)	7.0 (13.7)	8.0 (12.3)
15	6.0 (28.7)	7.5 (25.4)	9.0 (22.7)	10.5 (20.4)	12.0 (18.4)

Methods of interpolation have been discussed in section 6.

The tables may be used for $\gamma < 10$ by computing $N^* = N\gamma$ and finding the plan for N^* and $\gamma = 1$.

The tables on pp. 38 - 39 are based on the same assumptions with the only modification that the risks have been computed from Poisson probabilities. The fenctions $m = np_o$ and $M = Np_o$ have been tabulated for $M < 50{,}000$ with c and $r = p_1/p_o$ as arguments for $c \leq 99$ and $r = 0.10, 0.15, \ldots, 0.80$, and for $\gamma = 1$. The optimum plan is (c,m) for $M(c-1) < M < M(c)$.

For $\gamma < 10$ use $M^* = M\gamma$ and the table for $\gamma = 1$.

An approximation to the "binomial solution" may be obtained by using c from the Poisson table and correcting the corresponding n to $n_b = n - 1/3$ or by computing n_b directly as $n_b = (c + (2 - p_o)/3)/p_o$.

An auxiliary table of p_o as function of p_1 and $r = p_2/p_1$ has been given on pp. 40 - 41.

Single Sampling Tables for IQL = 0.5 per cent and γ =1

$100p_1$	0.10			0.15			0.20			0.25			0.30		
N	n	c	100P	n	c	100P	n	c	100P	n	c	100P	n	c	100P
100	All	-	-	All	-	-	All	-	-	All	-	-	All	-	-
200	138	0	87.1	138	0	81.3	138	0	75.9	138	0	70.8	138	0	66.1
300	138	0	87.1	138	0	81.3	138	0	75.9	138	0	70.8	138	0	66.1
500	138	0	87.1	138	0	81.3	138	0	75.9	138	0	70.8	138	0	66.1
700	138	0	87.1	138	0	81.3	138	0	75.9	138	0	70.8	138	0	66.1
1000	138	0	87.1	138	0	81.3	138	0	75.9	138	0	70.8	138	0	66.1
2000	138	0	87.1	335	1	90.9	335	1	85.5	335	1	79.5	138	0	66.1
3000	335	1	95.5	335	1	90.9	335	1	85.5	335	1	79.5	335	1	73.4
5000	335	1	95.5	534	2	95.3	534	2	90.7	534	2	84.9	534	2	78.3
7000	335	1	95.5	534	2	95.3	734	3	93.8	734	3	88.6	934	4	84.8
10000	534	2	98.3	734	3	97.4	734	3	93.8	934	4	91.2	1134	5	87.1
20000	734	3	99.3	934	4	98.6	1134	5	97.2	1534	7	95.8	1933	9	93.0
30000	734	3	99.3	934	4	98.6	1334	6	98.1	1733	8	96.7	2333	11	94.7
50000	734	3	99.3	1134	5	99.2	1733	8	99.1	2333	11	98.4	3133	15	96.9
70000	934	4	99.7	1334	6	99.5	1733	8	99.1	2533	12	98.7	3533	17	97.7
100000	934	4	99.7	1334	6	99.5	1933	9	99.4	2933	14	99.2	4133	20	98.4
200000	1134	5	99.9	1733	8	99.9	2333	11	99.7	3533	17	99.6	5133	25	99.2

Single Sampling Tables for IQL = 1.0 per cent and γ =1

$100p_1$	0.20			0.30			0.40			0.50			0.60		
N	n	c	100P	n	c	100P	n	c	100P	n	c	100P	n	c	100P
50	All	-	-	All	-	-	All	-	-	All	-	-	All	-	-
70	69	0	87.1	69	0	81.3	69	0	75.8	69	0	70.8	69	0	66.0
100	69	0	87.1	69	0	81.3	69	0	75.8	69	0	70.8	69	0	66.0
200	69	0	87.1	69	0	81.3	69	0	75.8	69	0	70.8	69	0	66.0
300	69	0	87.1	69	0	81.3	69	0	75.8	69	0	70.8	69	0	66.0
500	69	0	87.1	69	0	81.3	69	0	75.8	69	0	70.8	69	0	66.0
700	69	0	87.1	69	0	81.3	69	0	75.8	69	0	70.8	69	0	66.0
1000	69	0	87.1	167	1	91.0	167	1	85.5	167	1	79.6	69	0	66.0
2000	167	1	95.5	167	1	91.0	267	2	90.7	267	2	84.9	267	2	78.3
3000	167	1	95.5	267	2	95.3	267	2	90.7	367	3	88.6	367	3	81.9
5000	267	2	98.3	367	3	97.4	367	3	93.9	467	4	91.3	567	5	87.1
7000	267	2	98.3	367	3	97.4	467	4	95.9	667	6	94.7	767	7	90.5
10000	367	3	99.3	467	4	98.6	567	5	97.2	767	7	95.9	967	9	93.0
20000	367	3	99.3	567	5	99.2	767	7	98.7	1067	10	97.9	1366	13	96.0
30000	467	4	99.7	667	6	99.6	867	8	99.1	1167	11	98.4	1666	16	97.3
50000	467	4	99.7	667	6	99.6	967	9	99.4	1466	14	99.2	2066	20	98.4
70000	567	5	99.9	767	7	99.7	1067	10	99.6	1566	15	99.3	2266	22	98.8
100000	567	5	99.9	867	8	99.9	1167	11	99.7	1766	17	99.6	2566	25	99.2
200000	667	6	100.0	967	9	99.9	1366	13	99.8	2066	20	99.8	3166	31	99.6

Single Sampling Tables for IQL = 2.0 per cent and γ =1

$100p_1$	0.40			0.60			0.80			1.00			1.20		
N	n	c	100P	n	c	100P	n	c	100P	n	c	100P	n	c	100P
30	All	..	--	All	–	..	All	–	–	All	–	–	All	–	–
50	34	0	87.3	34	0	81.5	34	0	76.1	34	0	71.1	34	0	66.3
70	34	0	87.3	34	0	81.5	34	0	76.1	34	0	71.1	34	0	66.3
100	34	0	87.3	34	0	81.5	34	0	76.1	34	0	71.1	34	0	66.3
200	34	0	87.3	34	0	81.5	34	0	76.1	34	0	71.1	34	0	66.3
300	34	0	87.3	34	0	81.5	34	0	76.1	34	0	71.1	34	0	66.3
500	34	0	87.3	34	0	81.5	84	1	85.4	34	0	71.1	34	0	66.3
700	84	1	95.5	84	1	90.9	84	1	85.4	84	1	79.5	84	1	73.3
1000	84	1	95.5	84	1	90.9	133	2	90.8	133	2	85.1	133	2	78.5
2000	133	2	98.3	133	2	95.3	183	3	94.0	233	4	91.4	233	4	84.9
3000	133	2	98.3	183	3	97.5	233	4	95.9	283	5	93.3	333	6	89.1
5000	133	2	98.3	233	4	98.6	283	5	97.2	383	7	95.9	483	9	93.1
7000	183	3	99.3	233	4	98.6	333	6	98.1	433	8	96.8	583	11	94.8
10000	183	3	99.3	283	5	99.2	383	7	98.7	533	10	98.0	683	13	96.1
20000	233	4	99.7	333	6	99.6	483	9	99.4	683	13	99.0	933	18	98.0
30000	233	4	99.7	383	7	99.8	533	10	99.6	733	14	99.2	1083	21	98.6
50000	283	5	99.9	433	8	99.9	583	11	99.7	883	17	99.6	1283	25	99.2
70000	283	5	99.9	433	8	99.9	633	12	99.8	933	18	99.7	1433	28	99.4
100000	333	6	100.0	483	9	99.9	683	13	99.8	1033	20	99.8	1533	30	99.6
200000	383	7	100.0	533	10	100.0	783	15	99.9	1183	23	99.9	1833	36	99.8

Single Sampling Tables for IQL = 3.0 per cent and γ =1

$100p_1$	0.60			0.90			1.20			1.50			1.80		
N	n	c	100P	n	c	100P	n	c	100P	n	c	100P	n	c	100P
30	23	0	87.1	23	0	81.2	23	0	75.8	23	0	70.6	23	0	65.9
50	23	0	87.1	23	0	81.2	23	0	75.8	23	0	70.6	23	0	65.9
70	23	0	87.1	23	0	81.2	23	0	75.8	23	0	70.6	23	0	65.9
100	23	0	87.1	23	0	81.2	23	0	75.8	23	0	70.6	23	0	65.9
200	23	0	87.1	23	0	81.2	23	0	75.8	23	0	70.6	23	0	65.9
300	23	0	87.1	23	0	81.2	23	0	75.8	23	0	70.6	23	0	65.9
500	56	1	95.5	56	1	90.9	56	1	85.5	56	1	79.5	56	1	73.3
700	56	1	95.5	56	1	90.9	89	2	90.8	89	2	85.0	89	2	78.4
1000	56	1	95.5	89	2	95.3	89	2	90.8	122	3	88.8	122	3	82.2
2000	89	2	98.3	122	3	97.5	155	4	96.0	189	5	93.3	222	6	89.2
3000	89	2	98.3	155	4	98.6	189	5	97.3	255	7	96.0	322	9	93.1
5000	122	3	99.4	155	4	98.6	222	6	98.1	322	9	97.5	422	12	95.5
7000	122	3	99.4	189	5	99.2	255	7	98.7	355	10	98.0	455	13	96.1
10000	155	4	99.7	222	6	99.6	289	8	99.1	389	11	98.4	555	16	97.4
20000	155	4	99.7	255	7	99.8	355	10	99.6	489	14	99.2	722	21	98.7
30000	189	5	99.9	255	7	99.8	389	11	99.7	555	16	99.5	855	25	99.2
50000	189	5	99.9	289	8	99.9	422	12	99.8	622	18	99.7	955	28	99.5
70000	222	6	100.0	322	9	99.9	455	13	99.9	689	20	99.8	1055	31	99.6
100000	222	6	100.0	355	10	100.0	489	14	99.9	755	22	99.9	1155	34	99.7
200000	255	7	100.0	389	11	100.0	555	16	99.9	855	25	99.9	1322	39	99.9

Single Sampling Tables for IQL = 4.0 per cent and $\gamma = 1$

$100p_1$	1.20			1.60			2.00			2.40			2.80		
N	n	c	100P	n	c	100P	n	c	100P	n	c	100P	n	c	100P
30	17	0	81.4	17	0	76.0	17	0	70.9	17	0	66.2	17	0	61.7
50	17	0	81.4	17	0	76.0	17	0	70.9	17	0	66.2	17	0	61.7
70	17	0	81.4	17	0	76.0	17	0	70.9	17	0	66.2	17	0	61.7
100	17	0	81.4	17	0	76.0	17	0	70.9	17	0	66.2	17	0	61.7
200	17	0	81.4	17	0	76.0	17	0	70.9	17	0	66.2	17	0	61.7
300	42	1	91.0	42	1	85.5	42	1	79.5	42	1	73.3	17	0	61.7
500	42	1	91.0	67	2	90.7	67	2	84.9	67	2	78.3	67	2	71.1
700	67	2	95.3	67	2	90.7	91	3	89.0	91	3	82.4	91	3	74.9
1000	67	2	95.3	91	3	94.1	116	4	91.6	116	4	85.2	116	4	77.4
2000	91	3	97.6	116	4	96.1	166	6	95.0	216	8	92.2	241	9	85.8
3000	116	4	98.7	141	5	97.3	191	7	96.1	266	10	94.2	341	13	89.8
5000	141	5	99.3	191	7	98.8	241	9	97.6	341	13	96.2	466	18	93.1
7000	141	5	99.3	216	8	99.1	291	11	98.5	391	15	97.1	566	22	94.9
10000	166	6	99.6	241	9	99.4	316	12	98.8	466	18	98.1	691	27	96.4
20000	191	7	99.8	266	10	99.6	391	15	99.4	591	23	99.0	916	36	98.1
30000	216	8	99.9	316	12	99.8	441	17	99.6	666	26	99.3	1066	42	98.7
50000	241	9	99.9	341	13	99.9	491	19	99.7	766	30	99.6	1241	49	99.2
70000	241	9	99.9	366	14	99.9	541	21	99.8	841	33	99.7	1366	54	99.4
100000	266	10	100.0	391	15	99.9	591	23	99.9	916	36	99.8	1516	60	99.6
200000	291	11	100.0	441	17	100.0	666	26	99.9	1041	41	99.9	1766	70	99.8

Single Sampling Tables for IQL = 5.0 per cent and $\gamma = 1$

$100p_1$	1.50			2.00			2.50			3.00			3.50		
N	n	c	100P	n	c	100P	n	c	100P	n	c	100P	n	c	100P
30	14	0	80.9	14	0	75.4	14	0	70.2	14	0	65.3	14	0	60.7
50	14	0	80.9	14	0	75.4	14	0	70.2	14	0	65.3	14	0	60.7
70	14	0	80.9	14	0	75.4	14	0	70.2	14	0	65.3	14	0	60.7
100	14	0	80.9	14	0	75.4	14	0	70.2	14	0	65.3	14	0	60.7
200	33	1	91.2	33	1	85.9	33	1	80.1	33	1	74.0	33	1	67.8
300	33	1	91.2	33	1	85.9	33	1	80.1	33	1	74.0	33	1	67.8
500	53	2	95.5	53	2	91.0	53	2	85.3	53	2	78.8	53	2	71.7
700	53	2	95.5	73	3	94.1	73	3	89.0	93	4	85.2	93	4	77.3
1000	73	3	97.6	73	3	94.1	93	4	91.6	113	5	87.5	133	6	81.4
2000	93	4	98.7	113	5	97.4	153	7	96.1	193	9	93.3	233	11	88.0
3000	93	4	98.7	133	6	98.2	173	8	96.9	233	11	95.0	313	15	91.4
5000	113	5	99.3	153	7	98.8	213	10	98.1	313	15	97.1	433	21	94.5
7000	133	6	99.6	173	8	99.2	253	12	98.8	353	17	97.8	513	25	95.9
10000	133	[illegible]	99.6	193	9	99.4	273	13	99.0	413	20	98.6	613	30	97.2
20000	153	7	99.8	233	11	99.7	333	16	99.5	513	25	99.3	793	39	98.5
30000	173	8	99.9	253	12	99.8	373	18	99.7	573	28	99.5	913	45	99.0
50000	193	9	99.9	293	14	99.9	413	20	99.8	653	32	99.7	1053	52	99.4
70000	213	10	100.0	313	15	99.9	453	22	99.9	693	34	99.8	1153	57	99.5
100000	213	10	100.0	333	16	100.0	493	24	99.9	753	37	99.8	1273	63	99.7
200000	253	12	100.0	353	17	100.0	553	27	100.0	873	43	99.9	1473	73	99.8

Single Sampling Tables for IQL = 7.0 per cent and γ =1

$100p_1$	2.80			3.50			4.20			4.90			5.60		
N	n	c	100P	n	c	100P	n	c	100P	n	c	100P	n	c	100P
30	10	0	75.3	10	0	70.0	10	0	65.1	10	0	60.5	10	0	56.2
50	10	0	75.3	10	0	70.0	10	0	65.1	10	0	60.5	10	0	56.2
70	10	0	75.3	10	0	70.0	10	0	65.1	10	0	60.5	10	0	56.2
100	10	0	75.3	10	0	70.0	10	0	65.1	10	0	60.5	10	0	56.2
200	24	1	85.6	24	1	79.5	24	1	73.3	24	1	67.0	24	1	60.8
300	38	2	91.0	38	2	85.3	38	2	78.7	38	2	71.5	38	2	64.1
500	52	3	94.2	52	3	89.2	66	4	85.6	66	4	77.8	66	4	69.0
700	52	3	94.2	66	4	91.9	66	4	85.6	66	4	77.8	66	4	69.0
1000	66	4	96.2	95	6	95.1	109	7	91.1	109	7	83.4	109	7	73.3
2000	95	6	98.2	109	7	96.2	166	11	95.2	223	15	91.7	223	15	81.3
3000	109	7	98.8	152	10	98.2	209	14	96.8	266	18	93.4	366	25	87.1
5000	124	8	99.2	166	11	98.6	252	17	97.9	366	25	96.1	523	36	91.2
7000	138	9	99.4	195	13	99.1	266	18	98.2	423	29	97.1	623	43	93.0
10000	152	10	99.6	223	15	99.4	323	22	99.0	509	35	98.1	823	57	95.5
20000	166	11	99.7	266	18	99.7	409	28	99.5	623	43	98.9	1123	78	97.6
30000	195	13	99.9	295	20	99.8	423	29	99.6	723	50	99.3	1323	92	98.4
50000	209	14	99.9	323	22	99.9	509	35	99.8	823	57	99.6			
70000	223	15	99.9	323	22	99.9	523	36	99.8	866	60	99.7			
100000	238	16	100.0	366	25	99.9	566	39	99.9	966	67	99.8			
200000	266	18	100.0	409	28	100.0	652	45	99.9	1123	78	99.9			

Single Sampling Tables for IQL = 10.0 per cent and γ =1

$100p_1$	4.00			5.00			6.00			7.00			8.00		
N	n	c	100P	n	c	100P	n	c	100P	n	c	100P	n	c	100P
30	7	0	75.1	7	0	69.8	7	0	64.8	7	0	60.2	7	0	55.8
50	7	0	75.1	7	0	69.8	7	0	64.8	7	0	60.2	7	0	55.8
70	7	0	75.1	7	0	69.8	7	0	64.8	7	0	60.2	7	0	55.8
100	16	1	86.7	16	1	81.1	16	1	75.1	16	1	69.0	16	1	63.0
200	16	1	86.7	26	2	86.1	26	2	79.7	16	1	69.0	16	1	63.0
300	26	2	91.6	36	3	89.6	36	3	83.2	36	3	75.7	26	2	65.4
500	36	3	94.5	46	4	92.1	56	5	88.2	56	5	80.4	56	5	71.0
700	46	4	96.4	56	5	94.0	76	7	91.5	86	8	85.3	86	8	75.1
1000	56	5	97.6	76	7	96.4	96	9	93.8	116	11	88.7	126	12	79.2
2000	76	7	98.9	96	9	97.8	136	13	96.6	186	18	93.7	236	23	86.5
3000	86	8	99.2	116	11	98.6	156	15	97.4	226	22	95.3	326	32	90.2
5000	96	9	99.5	136	13	99.2	196	19	98.5	296	29	97.3	456	45	93.7
7000	106	10	99.6	146	14	99.3	216	21	98.9	336	33	97.9	546	54	95.3
10000	116	11	99.8	166	16	99.6	246	24	99.3	386	38	98.6	656	65	96.6
20000	136	13	99.9	196	19	99.8	296	29	99.6	486	48	99.3	876	87	98.3
30000	146	14	99.9	216	21	99.9	326	32	99.7	536	53	99.5			
50000	156	15	99.9	236	23	99.9	366	36	99.8	616	61	99.7			
70000	166	16	100.0	246	24	99.9	386	38	99.9	666	66	99.8			
100000	176	17	100.0	266	26	100.0	416	41	99.9	716	71	99.9			
200000	196	19	100.0	296	29	100.0	466	46	100.0	816	81	99.9			

Single Sampling Tables for IQL = 15.0 per cent and γ =1

$100p_1$	6.00			7.50			9.00			10.50			12.00		
N	n	c	100P	n	c	100P	n	c	100P	n	c	100P	n	c	100P
30	4	0	78.1	4	0	73.2	4	0	68.6	4	0	64.2	4	0	60.0
50	4	0	78.1	4	0	73.2	4	0	68.6	4	0	64.2	4	0	60.0
70	4	0	78.1	4	0	73.2	4	0	68.6	4	0	64.2	4	0	60.0
100	11	1	86.2	17	2	87.0	17	2	80.7	4	0	64.2	4	0	60.0
200	17	2	92.2	17	2	87.0	17	2	80.7	17	2	73.8	17	2	66.5
300	24	3	94.7	37	5	94.4	37	5	88.9	37	5	81.3	37	5	72.0
500	37	5	97.8	37	5	94.4	57	8	93.3	57	8	86.1	57	8	76.0
700	37	5	97.8	44	6	95.6	57	8	93.3	77	11	89.4	77	11	79.1
1000	37	5	97.8	57	8	97.5	77	11	95.8	97	14	91.8	117	17	83.8
2000	57	8	99.3	77	11	98.8	97	14	97.3	157	23	96.1	217	32	90.8
3000	57	8	99.3	77	11	98.8	117	17	98.3	177	26	96.9	277	41	93.3
5000	64	9	99.5	97	14	99.4	137	20	98.9	217	32	98.0	377	56	96.0
7000	77	11	99.8	104	15	99.5	157	23	99.3	257	38	98.8	437	65	97.0
10000	77	11	99.8	117	17	99.7	177	26	99.5	277	41	99.0	497	74	97.7
20000	97	14	99.9	137	20	99.9	217	32	99.8	337	50	99.5	657	98	98.9
30000	97	14	99.9	157	23	99.9	237	35	99.9	377	56	99.7			
50000	104	15	100.0	157	23	99.9	257	38	99.9	437	65	99.8			
70000	117	17	100.0	177	26	100.0	277	41	99.9	457	68	99.9			
100000	117	17	100.0	177	26	100.0	297	44	100.0	497	74	99.9			
200000	137	20	100.0	197	29	100.0	317	47	100.0	557	83	99.9			

Single Sampling Tables with Risk of 50 % for Lots of Indifference Quality

$B(c,m) = 0.50$, $r = p_1/p_o$, $m = np_c$, $M = Np_o$, $\gamma = 1$.

c	r m	0.80 M	0.75 M	0.70 M	0.65 M	0.60 M	0.55 M	0.50 M	0.45 M
0	0.693	16.8	14.2	12.5	11.3	10.5	10.0	9.65	9.46
1	1.678	25.0	21.6	19.5	18.3	17.5	17.3	17.4	17.9
2	2.674	32.5	28.7	26.5	25.4	25.1	25.4	26.6	28.7
3	3.672	39.6	35.6	33.6	32.9	33.3	34.9	37.9	42.9
4	4.671	46.4	42.4	40.8	40.8	42.4	45.9	51.8	61.5
5	5.670	53.5	49.6	48.4	49.5	52.8	58.9	69.1	86.3
6	6.670	60.3	56.7	56.4	58.8	64.4	74.1	90.6	119
7	7.669	67.5	64.3	65.0	69.1	77.6	92.2	117	163
8	8.669	74.1	71.7	73.7	80.0	92.1	113	150	220
9	9.669	81.5	79.8	83.3	92.3	109	139	192	296
10	10.67	88.7	88.0	93.3	106	128	168	243	396
11	11.67	95.6	96.1	104	120	149	203	307	527
12	12.67	103	105	115	136	174	245	386	702
13	13.67	111	114	127	153	201	294	485	932
14	14.67	119	124	140	172	232	351	607	1230
15	15.67	126	134	154	193	268	419	758	1630
16	16.67	135	144	168	215	308	500	946	2150
17	17.67	143	154	183	240	353	595	1180	2830
18	18.67	151	166	200	268	405	707	1470	3720
19	19.67	159	177	217	297	462	838	1820	4890
20	20.67	168	189	236	330	529	995	2260	6420
22	22.67	187	215	277	405	689	1390	3470	11000
24	24.67	206	243	323	496	893	1950	5320	18900
26	26.67	226	273	376	604	1160	2720	8120	32300
28	28.67	247	306	437	733	1490	3770	12400	55000
30	30.67	269	341	505	889	1920	5240	18300	
35	35.67	329	445	720	1430	3580	11800	53100	
40	40.67	397	574	1020	2270	6630	26300		
45	45.67	475	733	1420	3590	12200	58400		
50	50.67	563	930	1980	5640	22300			
60	60.67	778	1470	3790	13800				
70	70.67	1060	2300	7180	33100				
80	80.67	1420	3570	13500					
90	90.67	1890	5490	25100					
99	99.67	2440	8060	43900					

	r	0.40	0.35	0.30	0.25	0.20	0.15	0.10
c	m	M	M	M	M	M	M	M
0	0.693	9.45	9.60	9.98	10.7	11.9	14.0	18.6
1	1.678	18.9	20.6	23.4	28.1	36.6	54.0	101
2	2.674	32.2	37.8	47.1	63.9	98.2	183	490
3	3.672	51.0	64.6	88.9	137	250	594	2270
4	4.671	77.8	107	162	286	621	1680	10300
5	5.670	116	173	292	587	1520	5830	45600
6	6.670	171	276	518	1190	3670	17900	200000
7	7.669	250	438	912	2400	8800	54400	
8	8.669	361	688	1590	4790	20900		
9	9.669	520	1080	2770	9530	49500		
10	10.67	745	1680	4810	18900	116000		
11	11.67	1060	2610	8290	37100			
12	12.67	1520	4050	14300	73000			
13	13.67	2150	6260	24500				
14	14.67	3050	9660	41900				
15	15.67	4320	14900	71700				
16	16.67	6100	22800					
17	17.67	8600	35000					
18	18.67	12100	53600					
19	19.67	17000						
20	20.67	24000						
22	22.67	47100						

Table of $1000p_o$ (upper entry) and $100p_o$ (lower entry).

$$p_o = \left(\log\frac{q_1}{q_2}\right) \Big/ \left(\log\frac{p_2 q_1}{p_1 q_2}\right)$$

	$r = p_2/p_1$									
$100p_1$	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0
0.10	1.23	1.44	1.64	1.82	2.00	2.16	2.33	2.49	2.64	2.79
1.00	1.23	1.44	1.64	1.82	2.00	2.17	2.34	2.50	2.66	2.81
0.15	1.85	2.16	2.46	2.73	2.99	3.25	3.49	3.73	3.96	4.19
1.50	1.85	2.17	2.46	2.74	3.01	3.26	3.51	3.76	4.00	4.23
0.20	2.47	2.89	3.27	3.64	3.99	4.33	4.66	4.98	5.29	5.59
2.00	2.47	2.89	3.28	3.65	4.01	4.36	4.70	5.03	5.35	5.67
0.25	3.08	3.61	4.09	4.55	4.99	5.41	5.82	6.22	6.61	6.99
2.50	3.08	3.61	4.10	4.57	5.02	5.46	5.88	6.30	6.71	7.11
0.30	3.70	4.33	4.91	5.46	5.99	6.50	6.99	7.47	7.93	8.39
3.00	3.70	4.34	4.93	5.49	6.04	6.56	7.08	7.58	8.08	8.57
0.35	4.32	5.05	5.73	6.38	6.99	7.58	8.16	8.71	9.26	9.79
3.50	4.32	5.06	5.75	6.41	7.05	7.67	8.28	8.87	9.46	10.0
0.40	4.93	5.77	6.55	7.29	7.99	8.67	9.32	9.96	10.6	11.2
4.00	4.94	5.78	6.58	7.34	8.07	8.79	9.49	10.2	10.9	11.5
0.45	5.55	6.49	7.37	8.20	8.99	9.75	10.5	11.2	11.9	12.6
4.50	5.55	6.51	7.41	8.27	9.09	9.90	10.7	11.5	12.3	13.0
0.50	6.17	7.22	8.19	9.11	9.99	10.8	11.7	12.5	13.2	14.0
5.00	6.17	7.24	8.24	9.19	10.1	11.0	11.9	12.8	13.7	14.6
0.55	6.78	7.94	9.01	10.0	11.0	11.9	12.8	13.7	14.6	15.4
5.50	6.79	7.96	9.07	10.1	11.2	12.2	13.2	14.1	15.1	16.1
0.60	7.40	8.66	9.83	10.9	12.0	13.0	14.0	15.0	15.9	16.8
6.00	7.41	8.69	9.90	11.1	12.2	13.3	14.4	15.5	16.6	17.7
0.65	8.02	9.38	10.6	11.8	13.0	14.1	15.2	16.2	17.2	18.2
6.50	8.03	9.42	10.7	12.0	13.2	14.4	15.6	16.8	18.0	19.2
0.70	8.63	10.1	11.5	12.8	14.0	15.2	16.3	17.5	18.6	19.6
7.00	8.64	10.1	11.6	12.9	14.3	15.6	16.9	18.2	19.5	20.9
0.75	9.25	10.8	12.3	13.7	15.0	16.3	17.5	18.7	19.9	21.0
7.50	9.26	10.9	12.4	13.9	15.3	16.7	18.2	19.6	21.0	22.5
0.80	9.87	11.5	13.1	14.6	16.0	17.4	18.7	20.0	21.2	22.5
8.00	9.88	11.6	13.2	14.3	16.4	17.9	19.4	21.0	22.6	24.2
0.85	10.5	12.3	13.9	15.5	17.0	18.4	19.9	21.2	22.6	23.9
8.50	10.5	12.3	14.1	15.8	17.4	19.1	20.7	22.4	24.1	25.8
0.90	11.1	13.0	14.7	16.4	18.0	19.5	21.0	22.5	23.9	25.3
9.00	11.1	13.1	14.9	16.7	18.5	20.2	22.0	23.8	25.7	27.6
0.95	11.7	13.7	15.6	17.3	19.0	20.6	22.2	23.7	25.2	26.7
9.50	11.7	13.8	15.8	17.7	19.5	21.4	23.3	25.3	27.3	29.3
1.00	12.3	14.4	16.4	18.2	20.0	21.7	23.4	25.0	26.6	28.1
10.00	12.4	14.5	16.6	18.6	20.6	22.6	24.7	26.8	28.9	31.2

Table of $1000p_o$ (upper entry) and $100p_o$ (lower entry).

$$p_o = \left(\log\frac{q_1}{q_2}\right) \Big/ \left(\log\frac{p_2 q_1}{p_1 q_2}\right)$$

	$r = p_2/p_1$								
$100p_1$	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
0.10	2.79	2.94	3.09	3.23	3.37	3.51	3.65	3.78	3.91
1.00	2.81	2.96	3.11	3.26	3.41	3.55	3.69	3.83	3.97
0.15	4.19	4.41	4.63	4.85	5.06	5.27	5.47	5.68	5.88
1.50	4.23	4.46	4.69	4.92	5.14	5.36	5.58	5.80	6.02
0.20	5.59	5.89	6.18	6.47	6.75	7.03	7.30	7.57	7.84
2.00	5.67	5.98	6.29	6.60	6.90	7.20	7.50	7.80	8.10
0.25	6.99	7.36	7.73	8.09	8.44	8.79	9.13	9.47	9.81
2.50	7.11	7.51	7.91	8.30	8.69	9.07	9.46	9.84	10.2
0.30	8.39	8.84	9.28	9.71	10.1	10.6	11.0	11.4	11.8
3.00	8.57	9.06	9.54	10.0	10.5	11.0	11.5	11.9	12.4
0.35	9.79	10.3	10.8	11.3	11.8	12.3	12.8	13.3	13.8
3.50	10.0	10.6	11.2	11.8	12.3	12.9	13.5	14.1	14.6
0.40	11.2	11.8	12.4	13.0	13.5	14.1	14.6	15.2	15.7
4.00	11.5	12.2	12.9	13.6	14.2	14.9	15.6	16.3	17.0
0.45	12.6	13.3	13.9	14.6	15.2	15.9	16.5	17.1	17.7
4.50	13.0	13.8	14.6	15.4	16.1	16.9	17.7	18.5	19.3
0.50	14.0	14.8	15.5	16.2	16.9	17.6	18.3	19.0	19.7
5.00	14.6	15.4	16.3	17.2	18.1	19.0	19.9	20.9	21.8
0.55	15.4	16.2	17.0	17.8	18.6	19.4	20.2	20.9	21.7
5.50	16.1	17.1	18.1	19.1	20.1	21.1	22.2	23.3	24.4
0.60	16.8	17.7	18.6	19.5	20.3	21.2	22.0	22.9	23.7
6.00	17.7	18.8	19.9	21.0	22.2	23.3	24.5	25.8	27.1
0.65	18.2	19.2	20.2	21.1	22.0	23.0	23.9	24.8	25.7
6.50	19.2	20.5	21.7	23.0	24.3	25.6	27.0	28.4	29.9
0.70	19.6	20.7	21.7	22.7	23.8	24.8	25.7	26.7	27.7
7.00	20.9	22.2	23.6	25.0	26.5	28.0	29.6	31.2	32.9
0.75	21.0	22.2	23.3	24.4	25.5	26.5	27.6	28.6	29.7
7.50	22.5	24.0	25.5	27.1	28.7	30.4	32.3	34.2	36.2
0.80	22.5	23.7	24.9	26.0	27.2	28.3	29.5	30.6	31.7
8.00	24.2	25.8	27.5	29.2	31.1	33.0	35.1	37.4	39.9
0.85	23.9	25.2	26.4	27.7	28.9	30.1	31.3	32.5	33.7
8.50	25.8	27.6	29.5	31.5	33.6	35.8	38.2	40.9	44.0
0.90	25.3	26.6	28.0	29.3	30.6	31.9	33.2	34.5	35.7
9.00	27.6	29.6	31.6	33.8	36.2	38.7	41.6	44.9	49.0
0.95	26.7	28.1	29.6	31.0	32.3	33.7	35.1	36.4	37.7
9.50	29.3	31.5	33.8	36.3	39.0	42.0	45.5	49.7	55.7
1.00	28.1	29.6	31.1	32.6	34.1	35.5	36.9	38.3	39.7
10.00	31.2	33.5	36.1	38.9	42.0	45.6	50.0	56.2	88.8